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MANCHESTER COLLEGE OF SCIENCE AND TECHNOLOGY

Department of Textile Technology

MECHANICAL BEHAVIOUR OF TWISTED YARNS

1. Fatigue behaviour of twisted continuous filament yarns.

2. Geometric structure and form of yarns.

By

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U.S. Army Contract Number
DA-91-591-EUC-~~1467~~ 1833
01-4601-60

Final Technical Report. October 1961 - October 1962

"The research reported in this document has been made possible through the support and sponsorship of the U.S. Department of Army, through its European Research Office."

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(Report due: October 31st 1962)

A C K N O W L E D G E M E N T

The authors wish to express their thanks to Professor J. J. Vincent M.Sc.,F.T.I. for his interest in the work. Thanks are also due to Miss B. P. Stacey for help in preparing the diagrams and to Mr. D. J. Clarke for help and advice in the construction of apparatus used in the work.

Last but by no means least, we would like to record our gratitude to the European Research Office of the U.S. Department of Army for their generous support of the work.

SUMMARY

Part I

The design and construction of a versatile fatigue tester are discussed in considerable detail. Length and tension measurements and also break detection equipment are incorporated together with automatic recording facilities. A few results are presented but no theoretical approach has yet been made in their support.

~~Part II~~

The twisting of rubber strips ~~has been~~ was studied in order to shed further light on the "ribbon-twisted" form of twisted yarns. It ~~has been~~ was shown that two forms of structure occur - a twisted form and a wrapped form. The theory of the occurrence of these forms ~~has been~~ was worked out by considering the energy relations, and shown to agree with the experimental results.

C O N T E N T S

PART I : The fatigue properties of twisted continuous filament yarns.

P A R T I

THE FATIGUE PROPERTIES OF TWISTED CONTINUOUS FILAMENT YARNS

CHAPTER I

The Fatigue Tester

1.1 Introduction, design and purpose

In the last annual report* on this work, results with a five-station fatigue tester operating at constant length of stroke were given. The parameters recorded were the increase in length of the specimen (development of slack) during cycling, and the number of cycles to break. Because of the development of slack, the maximum stress occurring in each cycle fell rapidly and specimens frequently did not break even after 180,000 cycles, unless strokes very near to the breaking extension were used. During the present year, a new fatigue tester has been constructed, incorporating a mechanism for taking up slack together with a number of other improvements designed to give improved accuracy and increased information.

This instrument has advantages over the fatigue testers, such as the one described recently by Lyons**, because of the extent of the data which can be obtained. The instrument was designed to

* Mechanical behaviour of twisted yarns by A.J.Booth, O.N. Bose and J.W.S.Hearle, October, 1961, Final Technical Report, U.S.Army Contract DA-91-519-EUC-1467,01-4601-60

**W.J. Lyons. Textile Research Journal, 32, 448 (1962).

provide the following features:

(i) A range of sinusoidal extensions, capable of being imposed at various frequencies, and showing improved accuracy for short strokes ($<$ 10%). Other forms of extension could be imposed by special cams.

(ii) Provision for taking-up slack between each cycle of imposed extension. Mechanism can be rendered inoperative for fixed stroke cycling without take-up of slack.

(iii) Measurement of tension developed in each specimen throughout test.

(iv) Measurement of increase of length in specimen as a result of take-up of slack.

(v) Detection of break, indicating cycles to break on a counter.

(vi) Twenty stations, thus providing for testing of an adequate statistical sample on each run.

(vii) A multi-channel recorder and a switching mechanism to give automatic recording of data through the test.

(viii) Accuracy of measurement (to 3%); convenience in use to allow for carrying out a large testing programme in a reasonable time; versatility, so that the tester can be adapted to a variety of specimens and conditions of testing; protection from errors due to vibration; and robustness, to avoid any bending of parts as a result of the development of tensions of the order of 1 lb. in each of twenty specimens.

It has been found convenient to divide the description of the apparatus and of the methods adopted to satisfy the requirements for the tester into five sections:

(a) mechanical parts

(b) tension measurement

(c) length measurement

(d) yarn breakage detection

(e) automatic recording facilities

A schematic view of the tester showing the interrelation of various parts is shown in Figure 1, and a photograph of the tester in Figure 2.

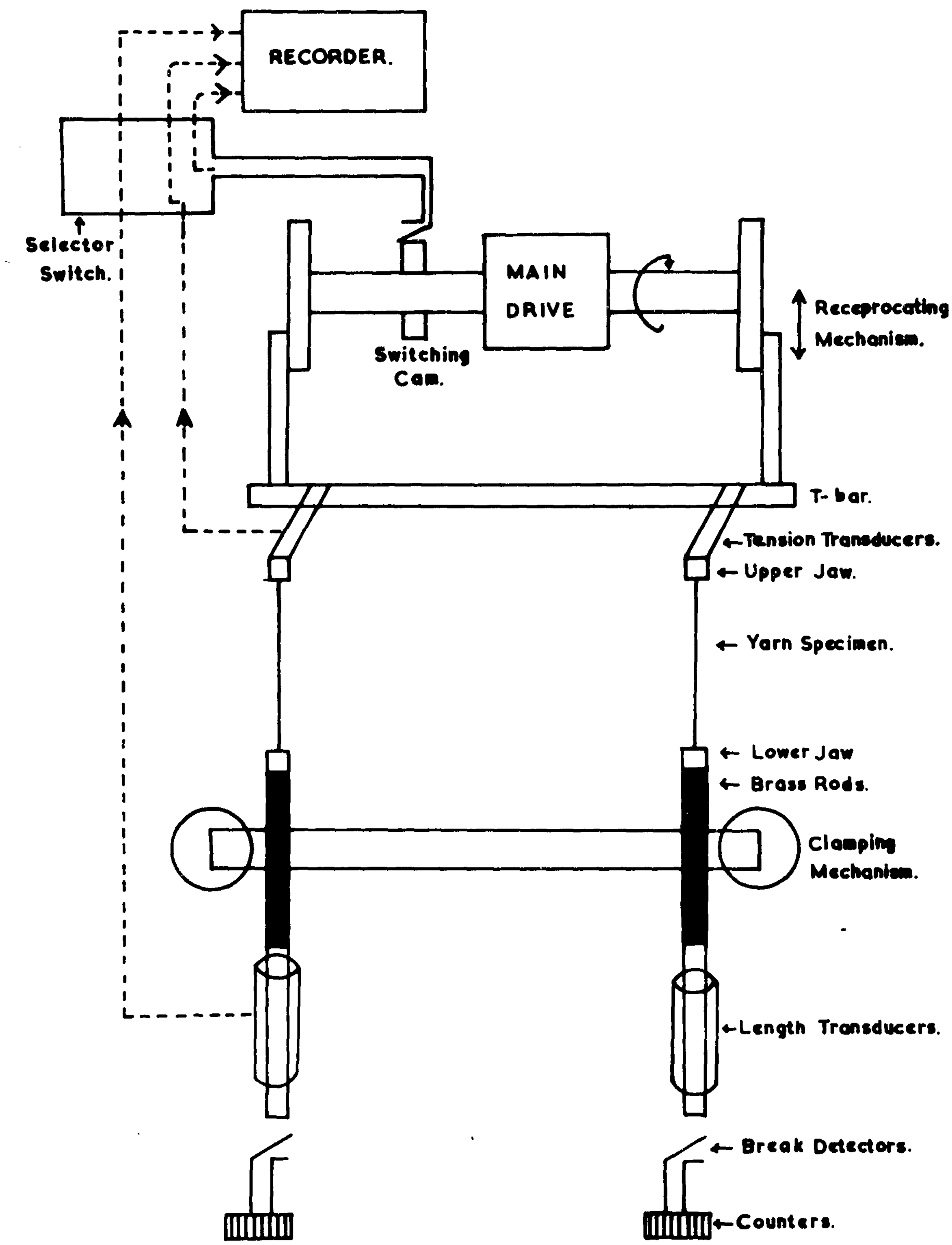
1.2 Mechanical parts

1.2.1 Basic motion

The basic essential of a fatigue tester is that it must be capable of producing a reciprocating motion over long periods of time. The simplest forms for this motion are those using the system of a crank and connecting rod to transmit angular motion into linear displacement.

With the Scotch yoke system (Figure 3(a)), the crank pin is constrained to move in a vertical slot at right angles to the line of motion and therefore since the reciprocating rod moves as the projection on the diameter of a point moving on the circumference of a circle, the uniform angular rotation of the crank is

FIG. I.



RECORDER.
Selector Switch.
MAIN DRIVE
Switching Cam.
Receprocating Mechanism.
T- bar.
Tension Transducers.
Upper Jaw.
Yarn Specimen.
Lower Jaw
Brass Rods.
Clamping Mechanism.
Length Transducers.
Break Detectors.
Counters.

transformed into simple harmonic reciprocation of the rod. Because only the fundamental vibration is present and the higher harmonics do not cloud the picture, simple harmonic motion is useful in calibrating vibration-measuring equipment. The important factor which limits the usefulness of this mechanism to relatively small machines operating under light loads is that the sliding block must transmit large forces to the reciprocating member by sliding contact.

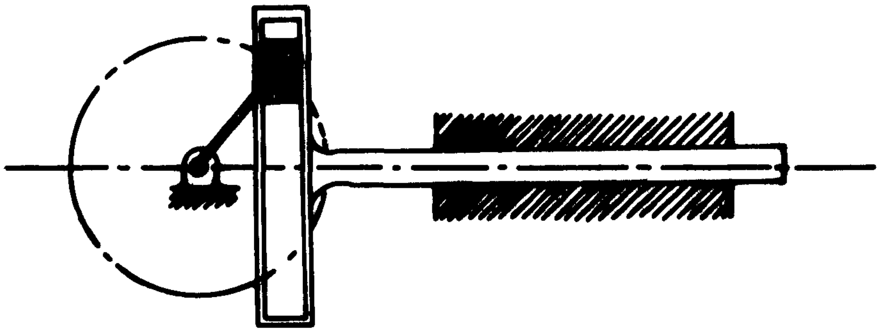
Due to these difficulties it was decided to use the simple crank and connecting rod, thereby sacrificing ideal simple harmonic motion. Since the slider-crank mechanism (Figure 3(b)) does not have an infinitely long connecting rod, a term known as the angularity of the system enters into the discussion and, as will be seen, the departure from S.H.M. depends on the ratio between the lengths of crank and connecting rod.

In Figure 3(b), if R is the length of the crank, L the length of the connecting rod and x the displacement of the slider from the extreme right hand position, i.e. top dead centre, then:

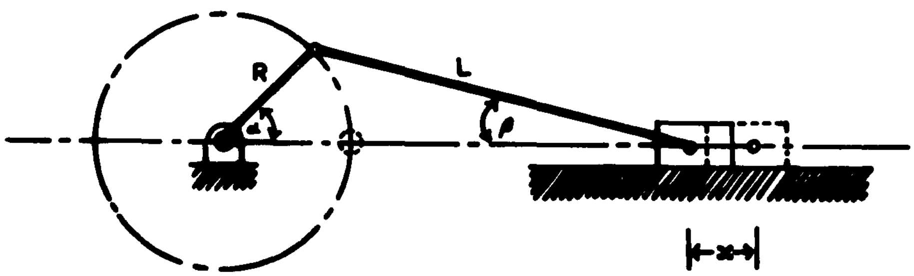
$$\begin{aligned} x &= R + L - R\cos\alpha - L\cos\beta \\ &= R(1-\cos\alpha) + L(1-\cos\beta) \\ &= R(1-\cos\alpha) + L\left(1-\sqrt{1-\left(\frac{R}{L}\right)^2\sin^2\alpha}\right) \end{aligned}$$

Expanding the quantity under the radical by the Binomial theorem,

FIG. 3.



(a) SCOTCH YOKE MECHANISM.



(b) SLIDER CRANK MECHANISM.

$$x = R(1 - \cos\alpha) + R\left[\frac{R}{2L}\sin^2\alpha + \frac{1}{8}\left(\frac{R}{L}\right)^3\sin^4\alpha + \frac{1}{16}\left(\frac{R}{L}\right)^5\sin^6\alpha + \ldots\ldots\ldots\right]$$

Since $x = R(1 - \cos\alpha)$ is the equation for the case of the infinitely-long connecting rod, the remaining terms take account of the angularity of the rod. Clearly as the ratio R/L becomes smaller the effect of angularity becomes insignificant, especially beyond the first term in the binomial expansion. When L = 20R, the increase in x due to angularity is 5% at 0°, 2.5% at 90°, zero at 180°, 2.5% at 270°, 5% at 360°. It can be seen from these figures that the deviation from S.H.M. takes the form of producing a curve which is unnoticeably dissimilar to S.H.M. between 90° and 270° and only slightly more inaccurate in the other two quadrants.

Having determined the nature of the reciprocating motion to be employed the rest of the machine was designed to fit in with it.

1.22 Moving-jaw assembly

For measurement of tension, strain gauges (to be described more fully in section 1.3) are mounted on ½" wide phosphor bronze strips, 3" in length. These strips, which carry the upper specimen jaws, are mounted in parallel as cantilevers from a 3 ft. length of 3/4" T-section steel bar, 1/8" thick. This material was chosen on account of its high resistance to bending, thus reducing considerably the errors due to bowing of a long bar.

This T-bar is supported at two points, 8" from either end by pairs of 3" mild steel tubes, ½" in diameter set 3" apart. These tubes slide in brass bearings, ½" in length mounted on stands fixed to the baseboard. Alignment is facilitated by a 1/4" steel bar connecting the two tubes on which are mounted two small brass pulleys which run in tracks and so provide smooth motion for the T-section bar. Bridged across the free end of the pairs of tubes is a 1/4" steel rod, from the centre of which the connecting rod, 8" in length, extends to the crank pin. The connecting rod contains a turnbuckle which serves to increase the length of the rod by up to 2" when necessary.

1.23 Main drive

It was decided with a view to the testing of heavy denier yarns that sufficient power should be available so that a high output torque should occur. A ½ h.p. motor with a reduction gear box, altering the speed of the output shaft from 1,420 down to 131 r.p.m. enabled this to be done satisfactorily. The motor is mounted on a bracket attached to the wall of the testing room and is above the main body of the tester. The separate mounting of this motor reduces vibration problems considerably. The output shaft carries a 4-step pulley, carrying a 3/8" V belt to the main shaft of the machine.

The main driving shaft is of ½" diameter mild steel bar and runs in 3 Oilite plummer blocks bolted to the baseboard. Keyed on

to either end of this shaft is a 2½" diameter steel disc. This disc has a milled slot, ½" wide, in which a brass strip 1/4" x ½" is free to slide. This strip is slotted either side of the centre to facilitate adjustment in eccentricity. A 3/16" silver steel pin is brazed to the centre of the strip and provides a bearing for the connecting rod mentioned above. A crank arm as such is therefore not employed directly.

Also mounted on the main shaft is a 4-step pulley and an auxiliary cam used for governing the clamping mechanism for the lower jaws (to be described later) and also for the operation of a microswitch in the revolution counter circuit.

This cam is designed so that the follower is raised by 3/16" for 30° of rotation of the mainshaft. As the follower is the nylon disc of a microswitch arm, this means that the electrical circuit employed with the clamping mechanism is closed for 330° of rotation.

1.24 Clamping mechanism

The lower specimen jaws were mounted at the top of brass rods, which could be clamped in position during the imposed extension but released to allow take-up of slack. Electromagnetic clamping was used. The three "Maxiflux" solenoid electromagnets have coil resistances of 2.6 ohms each and are connected in series. These are operated from a battery of 24v. This circuit contains a main switch and the microswitch operated from the cam mentioned above.

In parallel with the magnets is a resistance of 10^7 ohms and a galvanometer of the U.V. recorder. The function of this circuit is as a timing mechanism to show exactly when the clamps are energised and released. The high resistance used is to ensure that the current through the galvanometer is reduced to a low value. A "kick" of amplitude 1 cm. was found to be satisfactory and this was achieved. The solenoids have been exceptionally well designed and are capable of pulls of 8 lb. at 3 v. 1 A. and 200 lbs. at 24 v. and 3 A. Across the pole pieces of 3 electromagnets is fastened a 3 ft. length of steel bar 3/4" x 1/4". This is strengthened by a similar piece screwed at right angles to it. This addition was found to be necessary due to the bending which occurred with the single bar alone. This assembly constitutes the moving side of the clamping mechanism, the movement obtainable being 3/16". As can be seen from Figure 4, the inserts . finally chosen as being of useful friction properties without being too severe in their clamping action consist of two hard rubber bars, square in section, one of which is backed by rubber pressure tubing.

It was found that by this means, the 20 brass rods supporting the lower jaws could be held simultaneously with satisfactory reliability. Other methods of clamping which were tried but proved unsuccessful were:-

(i) Clamping directly on to the yarn.

(a) Silver steel to silver steel.

Due to the high accuracy of manufacture of silver steel it was considered possible that the grip upon several yarns simultaneously over a 3 ft. length would be virtually the same. This proved partially true but the severe action on the filaments of the yarn caused this method to be abandoned.

(b) Using soft rubber as a covering medium for the silver steel jaws.

This attempt to overcome the damage caused to filaments proved unsuccessful because it was found that the yarn was able to slip quite easily unless a very high clamp pressure was maintained. As this was impracticable over a 3 ft. length of bar, the method was discarded.

(ii) Clamping on to a metal insert holding the yarn.

(a) Using a shim brass strip.

The strip used was approximately 1/8" wide and 10 thousandths of an inch thick. It was found on trial that the silver steel clamping rods tended to bend and damage the strip so that mal-alignment of the lower yarn jaw was produced and also the free falling of the whole assembly was interfered with adversely.

(b) Using soft rubber strips as strengthening material for the above strips.

It was found that assistance was rendered by the addition of soft

rubber strips bonded to the brass strip by Araldite. Although bending of the strip was prevented to a slight degree, the frictional properties of the rubber strip were not sufficiently good and movement of the lower jaw due to the elastic yielding of the rubber proved unsatisfactory.

(c) Using silver steel clamping rods covered with rubber pressure tubing operating on brass rods.

This method proved very satisfactory from the point of view of non-slippage over a 3ft. length and was in fact used for some time but continual use produced grooves in the tubing caused by the pressure of the brass rods and eventually the rods began to adhere to one or other of the sides of the jaw.

(d) Using nylon clamping rods as jaw inserts.

The use of 3ft. lengths of nylon rod was envisaged as providing a hard wearing, friction surface for the brass rods but on trial, slip occurred at several positions due to long length of clamping required.

Using square section hard rubber bars, it was found that sufficient friction was obtained to clamp over a 3ft. length but also that the rubber yielded sufficiently to allow for slight variations in diameter of the brass rods, which is, of course, a factor of considerable importance. Being of hard rubber, the brass rods did not tend to adhere to the jaws but were completely freed as soon as the clamp opened and thus did not interfere at all with the accuracy of the length measuring system.

As is shown in Figure 4, a short length (1½") of soft iron rod is brazed on to the bottom of the brass rod as part of the length measuring system. The whole assembly is free to slide vertically in a glass tube, (chosen to minimise friction), which is prevented from falling through the coil by a rubber collar at its top end which rests on the coil end-cheek.

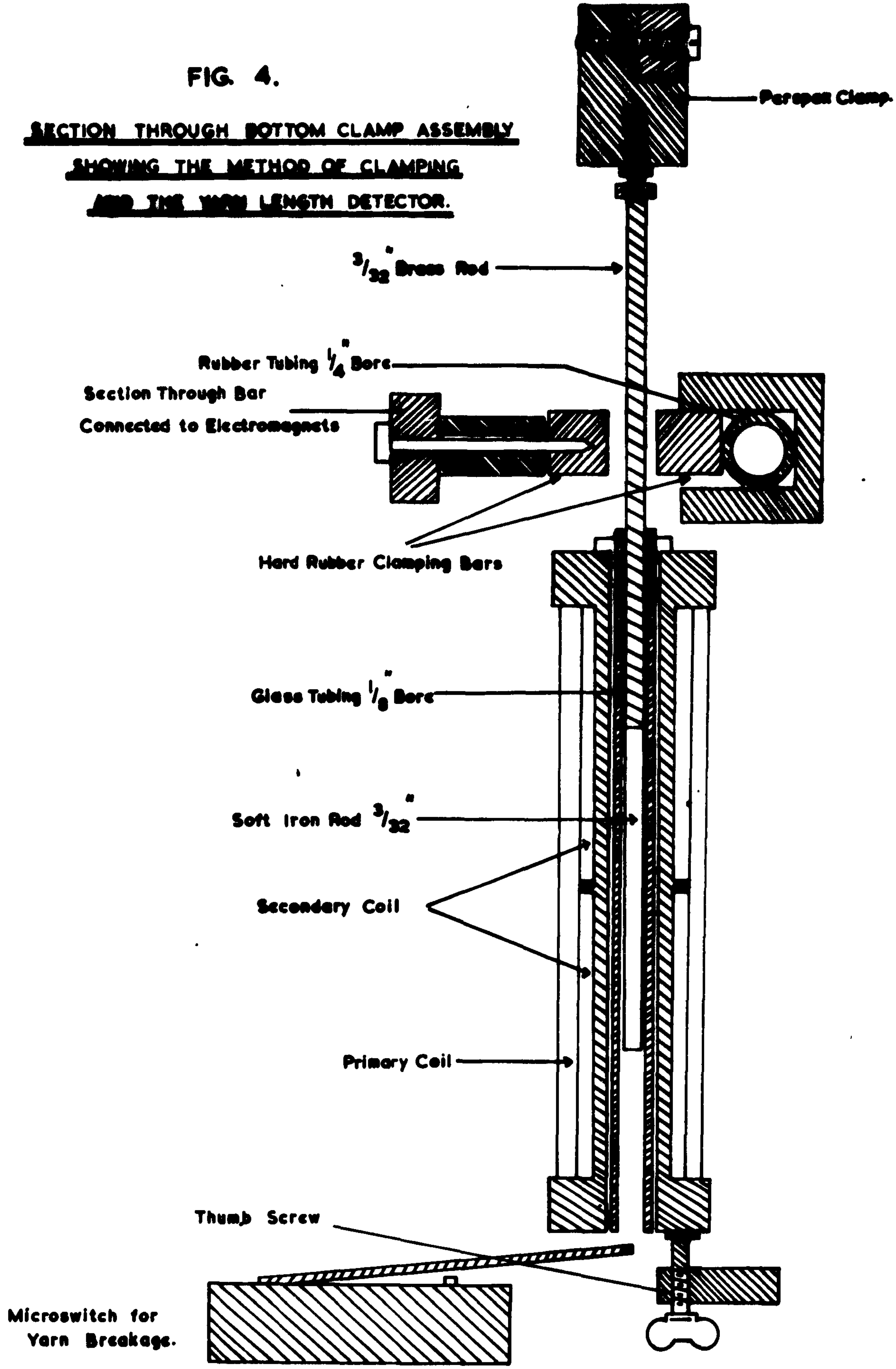
Unfortunately by using brass rod, which is sufficiently tough to withstand lateral pressure without deformation and by the necessity of using soft iron rod as a length measuring indicator the weight of the lower jaw assembly has risen to 8.5 gms. or 0.775 gms.wt./tex. This figure, however, does not compare unfavourably with the recommended figure of 0.5 gms.wt./tex. To decrease the weight of the lower jaw assembly to satisfy this condition could jeopardise the life of the assembly and also reduce the stability which it enjoys under the present circumstances.

1.3 Tension measurement

Electric resistance strain gauges are used as the active constituents in the measurement of stress developed in the yarn throughout the cycle. The etching process used in the manufacture of foil strain gauges produces a "ribbon" type conductor which is superior to a wire element of circular cross section both in electrical and mechanical properties. Width to thickness ratios in excess of 10 are obtainable giving better bonding to the backing

FIG. 4.

SECTION THROUGH BOTTOM CLAMP ASSEMBLY SHOWING THE METHOD OF CLAMPING AND THE YARN LENGTH DETECTOR.



material and better heat dissipation. The improved bonding gives a high gauge factor of the order of 2.05 - 2.20. The gauge factor of a strain gauge is a measure of the sensitivity and is defined as $dR/R \div dl/l$ where l is the length of wire in the strain gauge, R its resistance and the other quantities small increments in these two parameters. Another benefit of better bonding is that it allows higher shear forces to be transmitted, thus allowing a larger cross-section to be used than in the case of wire gauges and consequently increasing the current carrying capacity of the element. The improvement in heat dissipation also increases the current carrying capacity by preventing overheating.

In a ribbon element with a high width to thickness ratio, both the maximum shear stress and the surface temperature are reduced by as much as 50% compared to the circular wire gauge and with both these factors bearing a functional relation to gauge drift, the stability of the system is increased. The gauges employed in the machine are manufactured from cupro-nickel ribbon and have an epoxyethylene backing. Those used in the tests are the ½" linear foil gauge having a resistance of 80 ohms and a gauge factor of 2.20. The maximum permissible operating current depends on the heat-sink properties of the specimen, the material of which the specimen is made, the adhesive and type of gauge used and the quality of affixation; hence no fixed figure can be

accurately quoted. Typically the maximum permissible current may be between 100 and 650 mA. Gauges with epoxyethylene backing are limited to a maximum temperature of 100 degrees Centigrade.

Two of these gauges are mounted one on either side of the phosphor bronze strips referred to in 1.2. Phosphor-bronze was preferred to other materials on account of its low hysteresis properties. Beryllium-copper alloy would probably be even better but the expense incurred by its use does not seem warranted at the present time. The mounting of the gauges requires care and can cause errors if incorrectly performed. It is essential to have no air bubbles trapped between the gauge and the phosphor bronze strip. This ideal was best approached by carrying out the following procedure:

(i) after preliminary cleaning, mark the phosphor-bronze strip where the gauge is to be fixed.

(ii) slightly roughen the area to be covered with fine emery paper.

(iii) clean thoroughly with trichlorethylene

(iv) apply a small quantity of the prepared solution of strain-gauge cement and hardener 951 by spatula.

(v) lightly press the gauge on to the strip taking care that the whole of the underside of the gauge is coated with adhesive. Hard pressure at this point is not necessary and may indeed spoil the chances of a good bond.

(vi) remove the exuded cement by means of a sharp piece of cardboard or like material taking care that the position of the gauge is not disturbed. Although air bubbles may still now be present in the adhesive material these may be removed by the following step.

(vii) A piece of cardboard slightly larger in area than the gauge is placed over the gauge and gentle thumb and finger pressure over the whole area is sufficient to force trapped air bubbles out of the cement.

(viii) remove the exuded cement once more.

(ix) leave in this condition for 24 hours.

(x) remove all traces of cement solution from the gauge solder tabs. Apply a little Coralite flux and tin the tabs in the normal way. The strip and gauges are then ready to be connected to the machine and soldering of the leads is the only requisite.

(xi) using a small copper soldering iron, heated by a spirit burner, it was found possible to attach the leads successfully. The leads are .021 dia. single strand copper wire coated with P.V.C. It is essential to tin the leads before attempting to solder them to the tabs on the gauge. As little heat as possible is required and for this reason a low melting point solder is used. The melting point of the

solder is 145°C. and the recommended bit temperature is 185°C.

Two of these gauges constitute 2 arms of a Wheatstone bridge circuit. The other two arms of the bridge are formed by a pair of dummy gauges on a separate strip. As there are 20 pairs of active gauges and only one master dummy pair a switching circuit has been employed so that measurements from each bridge are taken consecutively. This is achieved by a two-bank 25 way Uniselector switch placed between the junction of the active gauges and the galvanometer. The galvanometer employed in the circuit has a sensitivity of 0.0018 mA per centimetre of scale deflection and has a resistance of 60 ohms.

The dimensions of the phosphor-bronze strip have been carefully chosen to take into account the bending required and deflection at the free end. Referring to Figure 5a :

Free length of beam (l) = 4.85 cms.

Clamped length = 2.08 cms.

Distance of middle of strain gauge from the fixed end (z) = 1.35 cms.

Width of phosphor-bronze strip (b) = 1.303 cms.

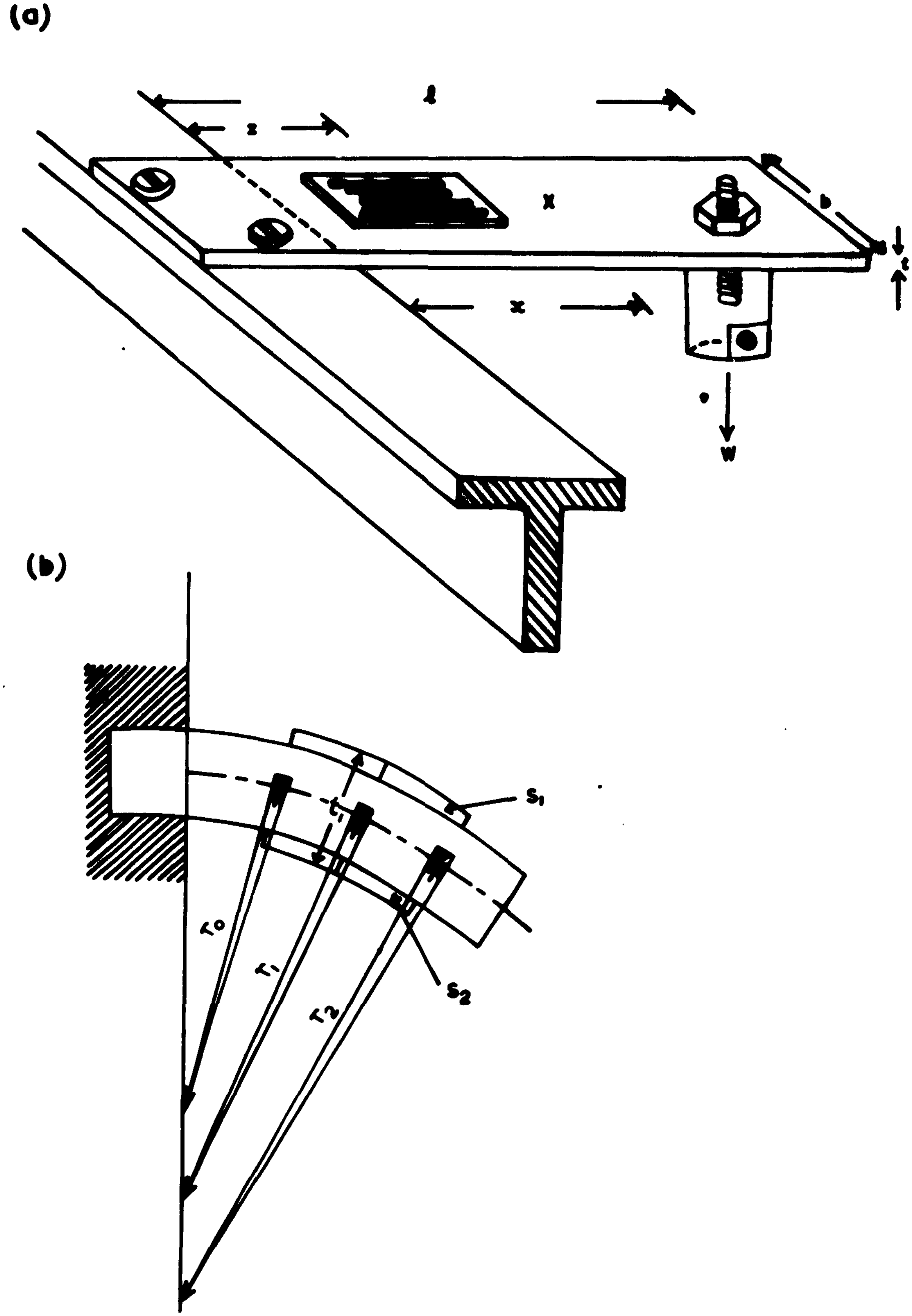
Thickness of strip (t) = 0.165 cms.

Thickness of strip where strain gauges are mounted = 0.201 cms.

Length of strain gauge = 1.27 cms.

The deflection y at any point X distant x from the fixed end is given by:

FIG. 5.



$$\frac{d^2y}{dx^2} = \frac{W(1 - x)}{EI}$$

Integrating w.r.t. x

$$\frac{dy}{dx} = \frac{W}{EI}(1x - \frac{x^2}{2}) + A$$

when $x = o$, $\frac{dy}{dx} = o$, therefore $A = o$

Integrating again

$$y = \frac{W}{EI}\left(\frac{1x^2}{2} - \frac{x^3}{3}\right) + B$$

When $x = o$, $y = o$, therefore $B = o$

therefore $y = \frac{Wx^2}{EI}\left(\frac{1}{2} - \frac{x}{6}\right)$

At the end of the beam, i.e. when $x = 1$

$$y = \frac{W}{EI} \frac{1^3}{3}$$

Therefore for a beam carrying 500 grams, the deflection at the point where the weight acts,

$$y = \frac{500 \times 981}{3 \quad EI} \times (4.85)^3$$

For the phosphor bronze used the figure for Young's Modulus given is 13.12×10^{11} dynes per sq. cm. The moment of inertia for the beam about an axis perpendicular to its length is $bt^3/12$

therefore $y = \frac{500 \times 981 \times 4.85^3 \times 12}{3 \times 13.12 \times 10^{11} \times 1.303 \times (.165)^3}$

$= \underline{0.02913 \text{ cms.}}$

This figure may seem excessive but it must be remembered that only nylon and Terylene yarns approach tensions of 500 grams. and this is reached at high extensions and thus the error involved is only 1.5% on a 2 cms. stroke or 0.29% if calculated on the initial gauge length.

The radius of curvature of the beam (r) is given by:

$$r = \frac{\left(1 + y_1^{\,2}\right)^{3/2}}{y_2}$$

where y_1 and y_2 are the first and second differentials of y with respect to x.

When y_1 is small (as it is here),

$$r = \frac{1}{y^2} = \frac{EI}{W(1 - x)}$$

As a guide to the length change occurring in the strain gauges it is necessary to calculate the radius of curvature at the point where the gauge is affixed. Here the combined effect of the small change in moment of inertia and modulus will give the beam added stiffness at this point, but in comparison with the stiffness of the phosphor bronze strip, any increase will be negligible. Therefore the modulus has been presumed to be the same for this section of the beam as for any other section and the moment of inertia has similarly been calculated for a thickness of beam of 0.165 cms. The radius of curvature for the bent strain gauge is calculated for the mid-point of the unbent gauge. Variation in the radius of

curvature over the ½" length of gauge will be discussed later. For the mid-point of the gauge the distance (z)(see Figure 5a) is 1.35 cms.

Considering the neutral axis of the beam with strain gauges mounted and a weight of 100 grams hung from the beam:

$$r_1 = \frac{13.12 \times 10^{11} \times 1.303 \times (0.165)^3}{100 \times 981 \times 3.5 \times 12}$$

$$= 1864 \text{ cms.}$$

where r_1 is the dimension shown in Figure 5b.

Assuming that this radius of curvature applies over the distance of ½", the neutral axis will remain ½" in length. The top strain gauge S, however, will increase in length and its new length

$$= (1864 + 0.1005)\ \frac{1.27}{1864}$$

$$= 1.27 + (6.846 \times 10^{-5}) \text{ cms.}$$

Similarly the new length of S_2

$$= 1.27 - (6.846 \times 10^{-5}) \text{ cms.}$$

therefore $\frac{dl}{l} = \frac{6.846 \times 10^{-5}}{1.27} = 5.391 \times 10^{-5}$

Now $\frac{dR}{R} = \lambda \frac{dl}{l}$ from the definition of gauge factor.

Here $\lambda = 2.20$

therefore $\frac{dR}{R} = 11.86 \times 10^{-5}$

When R = 80 ohms, the change in resistance of the top and bottom gauges is $\pm$ .009488 ohms respectively. This is the change for an

applied weight of 100 grams.

The Wheatstone bridge circuit shown in Figure 6 was used as the sensitive bridge for indicating this change in resistance. Resistances A and B are the active members of the bridge and C and D are the dummy pair. A_1 and B_1 are a second active pair (on a different phosphor bronze strip). Using Kirchhoff's second law for the closed circuit 1241, we have

$$Ai_2 - Gi_3 - C(i_1 - i_2) = 0$$

and for the circuit 2342

$$B(i_2 + i_3) + Gi_3 - D(i_1 - i_2 - i_3) = 0$$

For a particular circuit, the following measurements were taken: $i_1 = 320$mA; $i_3 = .0126$ mA. These results were found when the 100 gms. weight was added, the galvanometer scale deflection being 7.0 cms. The resistances of C and D remain constant with values of 80 ohms, whereas A, the top gauge, will increase in resistance by a small increment (dR) and the gauge underneath the phosphor bronze strip B will decreas by the same amount whe the beam is bent. The resistance of the galvanometer in the U.V. recorder is 60 ohms. Substituting these values in the equations above:

$$(80 + dR)\, i_2 - 60(.0126) - 80(320 - i_2) = 0$$

and $(80 - dR)(i_2 + .0126) + 60(.0126) - 80(320 - i_2 - .0126) = 0$

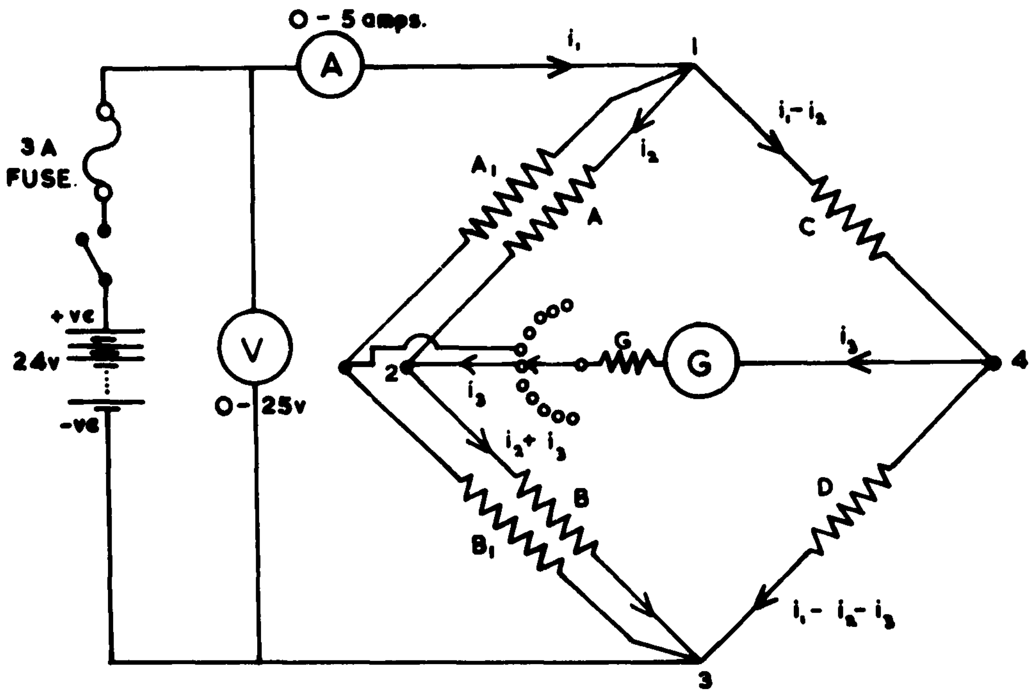
From the first of these equations

$$(160 + dR)i_2 - 25600 - 60(.0126) = 0$$

and from the second

FIG. 6.

WHEATSTONE BRIDGE CIRCUIT FOR STRAIN GAUGES.



$$(160 - dR)i_2 - 25600 + 220(.0126) - .0126dR = 0$$

Multiplying the first equation by (160 - dR) and the second by (160 + dR) and subtracting,

$$51200dR - 280.160.(.0126) + .0126(dR)^2 = 0$$

therefore

$$dR = \frac{-51200 \pm \sqrt{(51200)^2 + 640.280(.0126)^2}}{0.0252}$$

Taking the positive root,

$$dR = \frac{.0002778}{.0252} = .01103 \text{ ohms.}$$

therefore $\frac{dR}{R}$ (with R = 80 ohms) = 13.79×10^{-5}

This value is slightly higher than that obtained by the calculation of change in length. There are numerous assumptions which have been made in the calculations which would account for this slight discrepancy. Regarding the bent beam theory we have assumed that:-

i) the weight of the beam is negligible

ii) the curvature of the strain gauge is constant over its length of $\frac{1}{2}$"

iii) there is no slip of the strain gauge on the beam and that the beam is uniform over its length.

Of these, the second assumption is the most open to criticism because the radius of curvature varies between 1577 cms. at the fixed end to 2277 cms. at the end of the strain gauge further from the fixed end of the beam. To take the mean of these values (1927 cms.)

would, however, bring the value of dl even lower. The weight of the beam itself (20 gms.) however, if taken into account would have the effect of increasing the bending moment thereby decreasing the radius of curvature of the section of the beam to which the strain gauges are cemented.

Thus these two effects will tend to balance each other out, but the curvature error will be predominant. Other factors which enter the discussion of small errors are the changes in length and Young's Modulus for the phosphor bronze strip as the temperature rises to a final value of approximately 50°C. A rise of this order will produce a reduction in modulus of 0.08% and an increase in length of 0.01%.

Regarding the errors involved in calculating the change in resistance from the electrical circuit, the figure of 320mA given as the total bridge current may be inaccurate, due to the difficulty in measuring to better than 5mA. Also the galvanometer current given as 0.0126 mA could also be criticized because the deflection for 100 grams (7.0 cms.) is only accurate to 0.1 cms. The value of dR should probably be approximately .010 ohms, in other words a change of .012%. As can be seen from the theory of the bent beam the positioning of the strain gauge on the phosphor bronze is critical as regards the change in resistance of the gauge. The thickness of the cement layer also plays a minor part in determining

the sensitivity of the system. It can be understood therefore that the sensitivities of the 20 bridges may not be identical and they do in fact vary between 6.00 - 7.80 cms. for an added load of 100 grams.

The strain gauges used are designated as 80 $\pm$ 0.4 ohms. To balance the bridge initially, i.e. to have no current passing through the galvanometer, a means of zeroing and adjustment must be used. The technique employed was to position the galvanometer spot on the screen with the bridge disconnected and to watch the direction of movement of the spot when the switch was closed. Depending on this direction it was clear which of the pair of active gauges needed adjustment. With the connections employed, the top gauge needed adjustment if the spot moved to the left of zero. Jewellers' rouge, available in several degrees of abrasive power, was applied sparsely to the surface of the appropriate gauge by means of a calico cloth and slight rubbing, producing a minute change in resistance, brought the galvanometer spot back to its zero position.

This procedure was carried out on each of the 20 bridges in turn, the zero points for each bridge being attainable with great accuracy and comparative ease.

Approximately 160 mA passes through each strain gauge from a 24 volt battery. This means that in each gauge 3.84 watts is

dissipated. The majority of this energy is used in heating up the phosphor bronze strip and a check with a thermocouple showed that near the gauges the temperature of the strip was in the region of 50 - 60°C. This rise is undesirable but insufficient sensitivity is attained unless the amperage through the gauges is kept reasonably high. Such a high working temperature means that changes in ambient temperature caused by draughts can cause error. Changes in humidity can also cause:

a) breakdown of insulation between the gauge and earth

b) electrochemical corrosion of the gauge wire due to electrolysis which causes the gauge resistance to rise very considerably.

The effect of humidity changes may be summed up as causing zero drift.

<u>Calibration</u>

By depressing and then opening the switch in Figure 4, it can be seen whether the bridge is balanced correctly. This being so, calibration is effected by hanging known weights from the upper yarn jaw. A period of approximately 20 seconds was allowed between the weight being added and the reading on the galvanometer being taken. At the end of this period, oscillation of the weight has ceased and the beam is in a steady state. It is necessary to allow time for the strain gauges to reach a steady temperature before any attempt to calibrate is made. A period of 10 minutes after the

initial switching-on is considered as a minimum for this purpose. Although care is taken to minimise draughts and changes in humidity, minor errors due to these effects cannot be avoided but it is considered that serious errors are not involved. The possibility of coating the guages with strain-gauge cement as a waterproofing agent was considered but discarded due to the fact that the maximum operating current would be reduced by 25%.

The response of the bridge has been found to be reliable with a good linear relationship between weight applied and output current or scale deflection. It would appear that the bridge is slightly more sensitive in the range 0 - 100 gms. than above this range, and this is an added advantage. The reason for the slight increase in sensitivity in the lower ranges is ascribed to the fact that at or near balance, any bridge errors tend to cancel out, whereas in the higher ranges they will add together.

A typical calibration curve is shown in Figure 7.

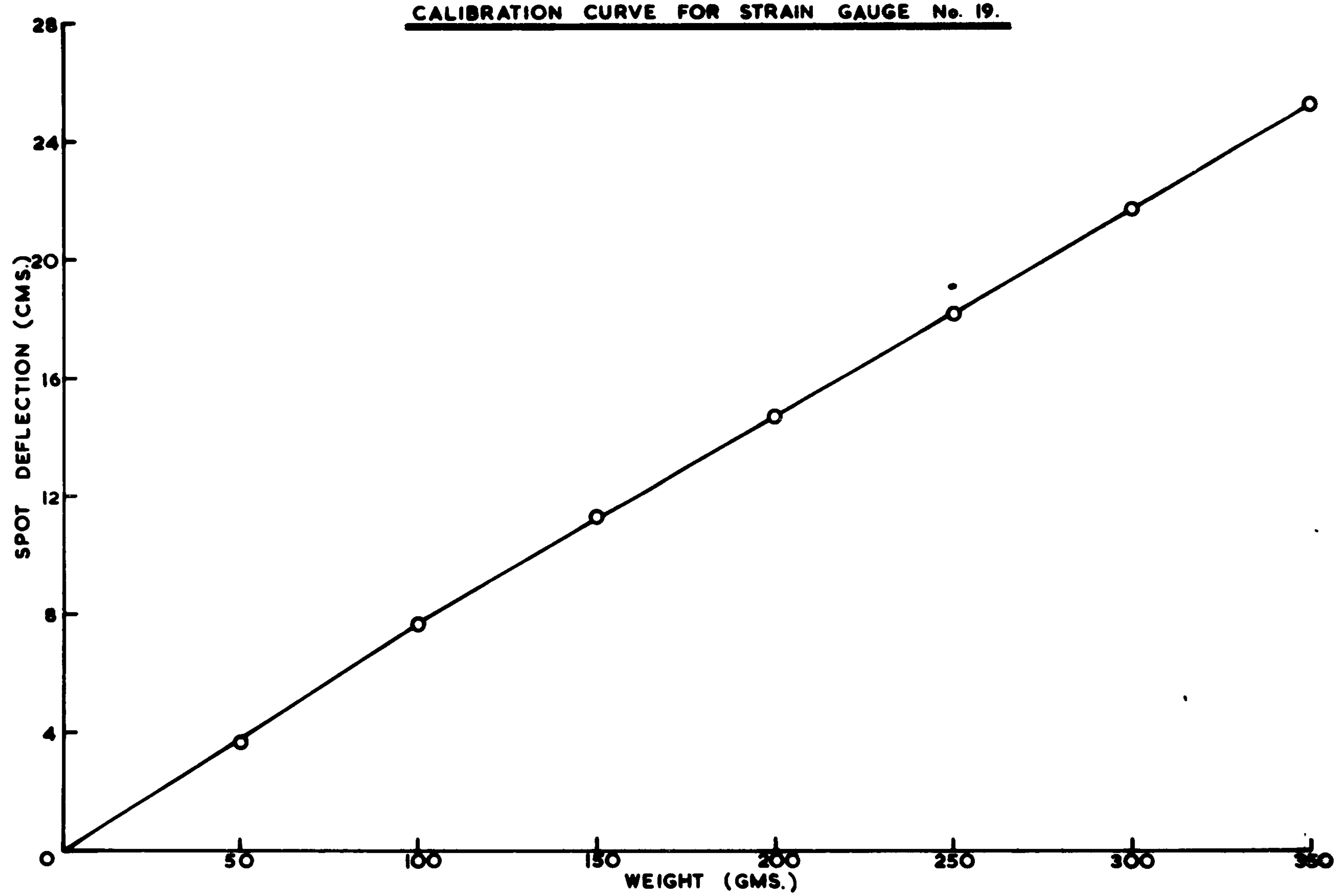
1.4 Instantaneous length measurement

1.41 Simple theory of an electromagnetic transducer

In order to measure accurately the increases in length of the yarn specimen, an electromagnetic transducer taking the form of a differential transformer has been used. It takes advantage of the fact that the mutual inductance between two coils changes as the permeability of the medium in their vicinity alters.

FIG. 7.

CALIBRATION CURVE FOR STRAIN GAUGE No. 19.



When a current is passed through the primary coil A having an inductance L_1 (see Figure 8), an E.M.F. is induced in the two secondary coils B and C (of equal inductance L_2). The magnitude of the induced E.M.F. in the secondary coil depends upon the inductances of the primary and secondary coils, their proximity, the permeability of the medium of the coil centre and the rate at which the primary current changes. All these factors except the rate of change of primary current are grouped together and the combination is called the mutual inductance of the circuit.

Suppose that coil A has N_1 turns and coil B has N_2 turns and that the coils are so close together that the whole of the flux produced by the current in A links with coil B. Let this flux be ϕ when the current in coil A is i_1. Then the self-inductance of coil A is $L_1 = N_1 \frac{\phi}{i_1}$ and the mutual inductance M is given by

$$M = N_2 \frac{\phi}{L_1} = \frac{N_2}{N_1} L_1.$$

Similarly if i_2 flows in coil B, its self inductance $L_2 = N_2 \frac{\phi}{I_2}$

and $M = N_1 \frac{\phi^1}{L_2} = \frac{N_1}{N_2} L_2.$

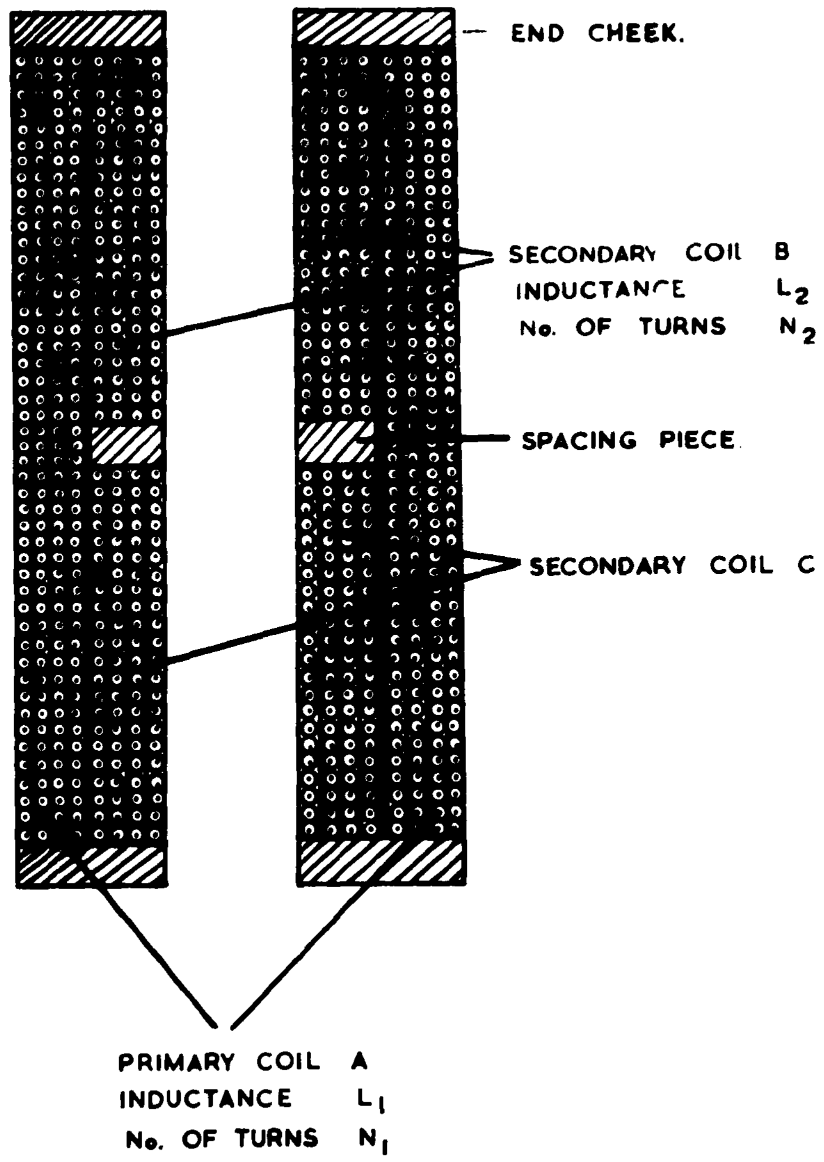
Therefore $M = \frac{N_2}{N_1} . L_1 = \frac{N_1}{N_2} L_2$

therefore $M^2 = L_1 L_2$ and $M = \sqrt{L_1 L_2}$

It has been assumed that the flux linkages are complete but in practice it is found that a factor k, less than unity, has to

FIG. 8.

SECTION THROUGH DIFFERENTIAL TRANSFORMER.



be introduced on the right hand side of the equation. For air-cored coils k is of the order of 0.05 whereas with completely iron-cored coils k can rise to 0.98. Thus it becomes clear that as the soft iron bar enters the coil assembly, the flux linkages between the two coils increases from a low to a high figure as the permeability of the medium increases. The increase in flux linkages produces a corresponding increase in the mutual inductance and therefore a change in the induced voltage in the secondary coil. The induced voltage reaches a maximum when the bottom of the iron bar reaches the bottom of the top coil B. As the bar enters the lower secondary coil C the mutual inductance of this coil begins to rise and that of the upper coil to decrease, because the length of the iron rod is equal to the length of the secondary coil. A point is reached at which the mutual inductances of the coils are equal and the induced voltages are of equal magnitude. Since the two secondary coils are connected in series opposition rather than series aiding the induced currents are in opposite directions and so the galvanometer will read zero rather than double the current produced in one of the coils. As the bar progresses down the centre of the lower coil thereby leaving the top coil, the current in the galvanometer will again begin to rise to a maximum. At the balance point there is a 180° phase change and this is clearly seen on the galvanometer scale.

1.42 Design and construction of a suitable coil

The following requirements were borne in mind when the design of the coil was contemplated.

a) Maximum sensitivity over as large a range of displacement as possible.

b) Stability to temperature change.

c) Outside diameter 1" (limited by distance between stations).

d) Ability to be easily mounted in a 20 station rack.

In designing inductance coils, it is found that for most purposes a high time constant is desirable (in other words a high value for inductance and a low value for resistance). For any given length and gauge of wire, the resistance is a fixed quantity but the inductance can be greatly varied depending upon what shape of core is used, the spacing between turns and the number of turns used. In general the closer together the windings and the greater the number of turns on a coil, the greater will be its inductance. Therefore for a given length and diameter of coil it would seem advisable to use very thin wire and accommodate as many turns as possible. However, the resistance of a wire increases appreciably as its diameter is reduced and so a compromise must be attained. From the point of view of the number of lines of force between the primary and secondary coils, the closer the turns are wound, the

more sensitive will be the coil to the insert of the rod.

One factor which must not be overlooked is the current carrying capacity of the coil. The steps in the determination of coil dimensions are therefore as follows:

a) determine the maximum voltage and amperage under which the coil will be called upon to operate, V_{MAX} and I_{MAX} .

b) having determined I_{MAX}, look up the S.W.G. for the wire in question which has a current carrying capacity equal to or greater than I_{MAX}.

c) having chosen the S.W.G. for the wire, find out the resistance in ohms per metre for this wire.

d) from $\frac{V_{MAX}}{I_{MAX}}$, the minimum permissible resistance of the coil in ohms can be established.

e) from this value of resistance and from the resistivity in ohms per metre, the minimum length of wire can be calculated.

f) for a given length and diameter of coil former, this means that the number of coil layers can be calculated assuming that the winding of consecutive turns is as close as possible.

Dimensions such as the thickness of any insulating coating the wire may have and the thickness of separating material between layers must be taken into account. In the coils used, eureka wire was chosen on account of its low temperature coefficient of resistance in preference to copper which it was feared would be subject

to too much fluctuation in temperature and differential heating. The wire chosen for both primary and secondary coils was 30^{s}S.W.G. (dia. 0.0124") with an enamel insulation coating. The length of the former, made of tufnol, between the end cheeks is 2.3/4". A spacing piece to separate the secondary coils in the middle is 1/32" thick. The diameter of the former is 3/8" with a bore of 1/4". The end cheeks are 1/4" long and 3/4" diameter.

The resistance of eureka wire is 6.28 ohms per metre and the resistance of the secondary coil was 100 ohms. Therefore 15.92 metres of wire were used. This was coiled in three layers of 150 turns each with a layer of thin tissue paper between layers. For the primary coil 1000 turns of 30^{s} S.W.G. eureka wire were wound in 3 layers immediately over the secondary coils. This coil has a resistance of 360 ohms and a depth of winding equal to or slightly less than that of the secondaries.

The self inductance of the coils as measured on a Marconi Universal Bridge, was found to be 1.65 millihenries for the secondaries and 5.6 millihenries for the primary coil. The mutual inductance between the primary and one secondary, found by taking 1/4 of the difference between the combined inductance series aiding and series opposition, is estimated to be 50 - 100 microhenries and that between the two secondary coils to be 10 or 20 microhenries. All these recordings are for the coil with air as the permeable medium.

Throughout the design of the coil assembly the weight of the soft iron rod has had to be kept to a minimum so that the initial tension on the yarn may be kept as low as possible and also that the impact forces imposed upon the yarn as the bottom yarn jaw falls under gravity may be minimised. Should the need ever occur that the lower yarn jaw assembly be reduced appreciably in weight while still maintaining a similar sensitivity, it is possible that mumetal, possessing high permeability and low hysteresis values, could be used. Unfortunately the minimum diameter of mumetal rod commercially available at the time of use was 1/4" and this was found to be extremely difficult to machine to a lower diameter successfully.

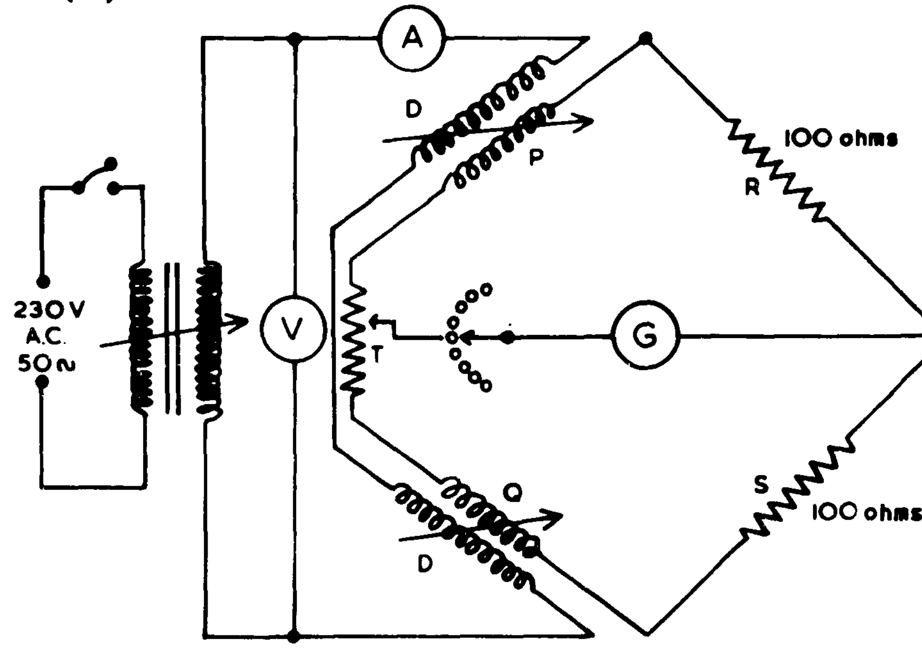
1.43 Method of measurement

To measure the out of balance current, a modified form of the Wheatstone bridge is used. (Figure 9). In Figure 9a, the two secondary coils are represented by P and Q and the primary coil (shown split) by D. Between the two secondaries is a 10 ohm variable potentiometer T. From the central terminal of this potentiometer a lead is taken to 1 terminal of the uniselector switch and from the latter's common terminal to the galvanometer G of the U.V. recorder. In the other two arms of the bridge are standard 100 ohm resistances. Dubilier high stability resistors with a tolerance of ± 1% were used and found satisfactory. The

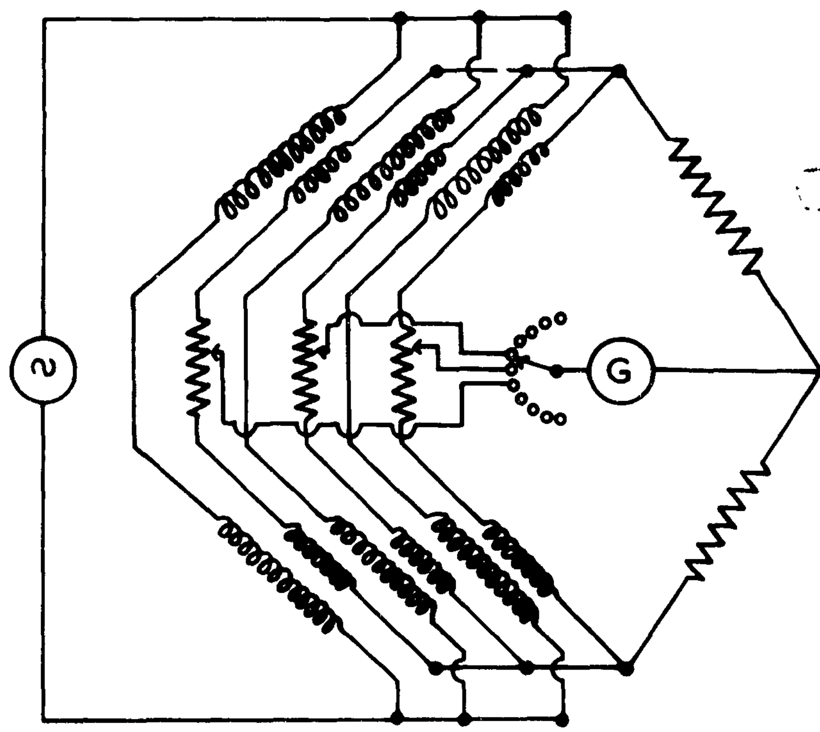
FIG. 9.

THE MODIFIED INDUCTANCE BRIDGE CIRCUIT.

(a)



(b)



primary coil D is fed from a variac which is connected through a switch to the mains (230v. A.C. 50 cycles). A voltmeter reading to 25 volts A.C. is connecte across the primary coi and an ammeter reading to 1.5A. A.C. is in series with the coils. The arrows placed across the active arms of the bridge represent the variable mutual inductance caused by the change in position of the soft iron rod.

With the machine having 20 stations it is necessary for each position to have one of these electromagnetic transducers and so the 20 positions are connected in parallel and the one pair of resistors R and S are used as the other arms for each bridge. To illustrate the circuit three sets of coils are shown connected in Figure 9b. It is clear that although every bridge is being excited all the time the out of balance current from one bridge only can be measured at any one time.

1.44 Calibration procedure

Before the soft iron rod is inserted into the coil assembly it is essential to check that there is zero current passing through the galvanometer to ensure that the bridge is balanced. If there is any deflection it means that the ratio of the resistances of the coils P and Q is not equal to that of the standard resistors R and S (which may not be exactly unity). This can be remedied by altering the appropriate potentiometer T slightly. Unfortunately, however, this procedure does not balance the difference in inductance (if any exists) between the secondary coils. Any

unbalance is accentuated by increase in voltage and therefore it has been found practical to use 15 v. when this procedure is being carried out so that fine balance can be achieved. As the working voltage is only 6 v. any unbalance of inductance is negligible. Any differences in capacitive reactances, which are in any case low at 50 cycles frequency, have been neglected.

When the coils have been balanced electrically it is then necessary to adjust the overall position of the coil assembly with respect to the soft iron rod. As the gauge length for the yarn is 10 cms. and this is a fixed amount with respect to the top jaw position, the middle point of the soft iron rod must coincide with the centre of the coil assembly when the rod is inserted. This ensures an initial position where the yarn is 10 cms. long and the system is balanced. This zero adjustment is carried out by turning the thumb-screw (refer to Figure 4) which bears upon a small piece of brass shim cemented to the bottom of the lower coil cheek. One turn of this screw produces a difference of 0.025" in the coil height. The cheeks of the coil are held by strong spring clips so that although the thumb screw requires a considerable torque to move the coil, once the zero position has been located it is extremely unlikely that the coil will slip, causing readjustment to be called for.

Calibration of the coils is achieved with the help of a depth

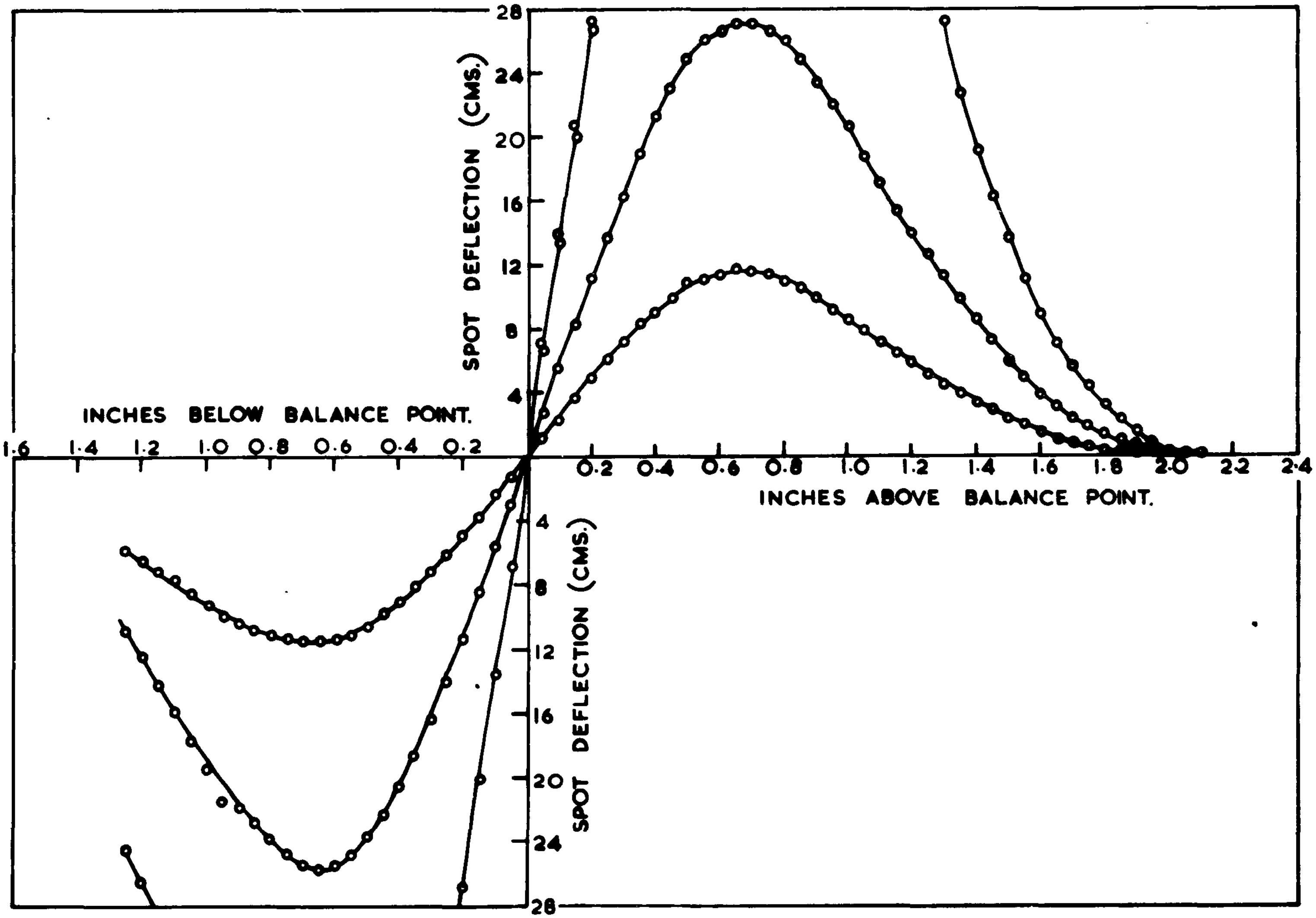
micrometer capable of measurable movements of 0.00025". To the end of the moving head is locked a stud which supports a 4" long rectangular brass bar, projecting at right angles to the axis of the micrometer. From this bar, another small brass rod, with a narrow flat end portion fits into the lower yarn jaw, which holds the insert for the coil assembly. The micrometer itself is held rigidly in a clamp fastened to a retort stand. The 4" brass bar is able to be rotated through 360^{o} and this fact facilitates the alignment of the micrometer head and the yarn jaw. The amount of looseness of the bar is in the region of 0.00012" at the free end which is considered negligible when measurements in stages of 0.025" are being taken.

The calibration curve for one station is shown in Figure 10. The head of the micrometer was moved downwards in steps of 0.050" and the reading on the scale of the U.V. recorder taken. As it is an A.C. signal which is being measured, the reading taken is twice the amplitude of the A.C. wave.

Taking the 6 v. curve it is clear that as the iron rod enters the upper secondary coil the flux linkages begin to increase slowly at first, then at a steady rate until they reach a maximum when the iron core is totally in the upper coil. The initial slow rise is probably due to there being a considerable loss in the number of lines of force near the top of the coil which actually link with the secondary coil. As the iron rod progresses into the lower coil

FIG. 10.

CALIBRATION CURVE FOR MUTUAL INDUCTANCE SYSTEM No. 20



it is evident that the transducer shows an improvement in both sensitivity and linearity. Both these facts are probably due to there being little or no flux loss near the centre of the coil assembly. Within 0.3" on either side of balance there is a good linear relationship, any movement of the rod being magnified 21 times approximately.

By increasing the voltage to 12 v. the output current is increased twofold but the galvanometer scale is not large enough to contain the whole movement of the rod as the yarn extends to break.

By decreasing the voltage to 3 v., full use is not being made of the 28 cms. galvanometer scale.

If it is known that the yarn under test is not likely to extend very much (for example nylon or Terylene at low stroke values), more accurate measurements of change in length can be made on the 12 v. scale but for most measurements the 6 v. scale is used. The discontinuity of the curve in the lower left hand corner of the calibration curve is explained by the fact that as the iron rod leaves the coil at the bottom, the lower yarn jaw comes in contact with the electromagnetic clamp and so the rod is not allowed to leave the coil completely.

It has been assumed that it is advisable to start the test from the balance point and only use the portion of the curve below this point but to make full use of the linear portion of the curve it is better to start at a point 0.3" above balance. Using this system,

however, there is the disadvantage that near the balance point, uncertainty arises as to which side of balance is being used and there is no foolproof method of ascertaining this.

For yarns such as viscose rayon and acetate rayon which have a large initial extension during the first 100 cycles, this method can be profitably used.

Due to small discrepancies between the sets of coils and the difference in permeability of the soft iron rods it is necessary to calibrate each coil assembly separately, thus necessitating 20 calibration curves. It has been found, however, that the maximum deflection of the galvanometer spot varies between 26 and 29 cms. on the 6 v. scale and it is felt that this is acceptable considering the many unknown factors prevailing. Among those factors may be mentioned eddy current losses and the possibility of considerable variation in permeability from one rod to the next.

1.5 Automatic break detection

In order to obtain a satisfactory statistical sample of breaking times, it is necessary for each yarn to be equipped with some form of break detector. As mentioned in the last annual report it is unadvisable to actually touch the yarn because this involves the possibility of damage to the surface filaments which can result in premature fatigue. To use photo-electric cells on each yarn is expensive and also open to error due to the thin beam of light required. Due to the necessity to remove any slack produced in the

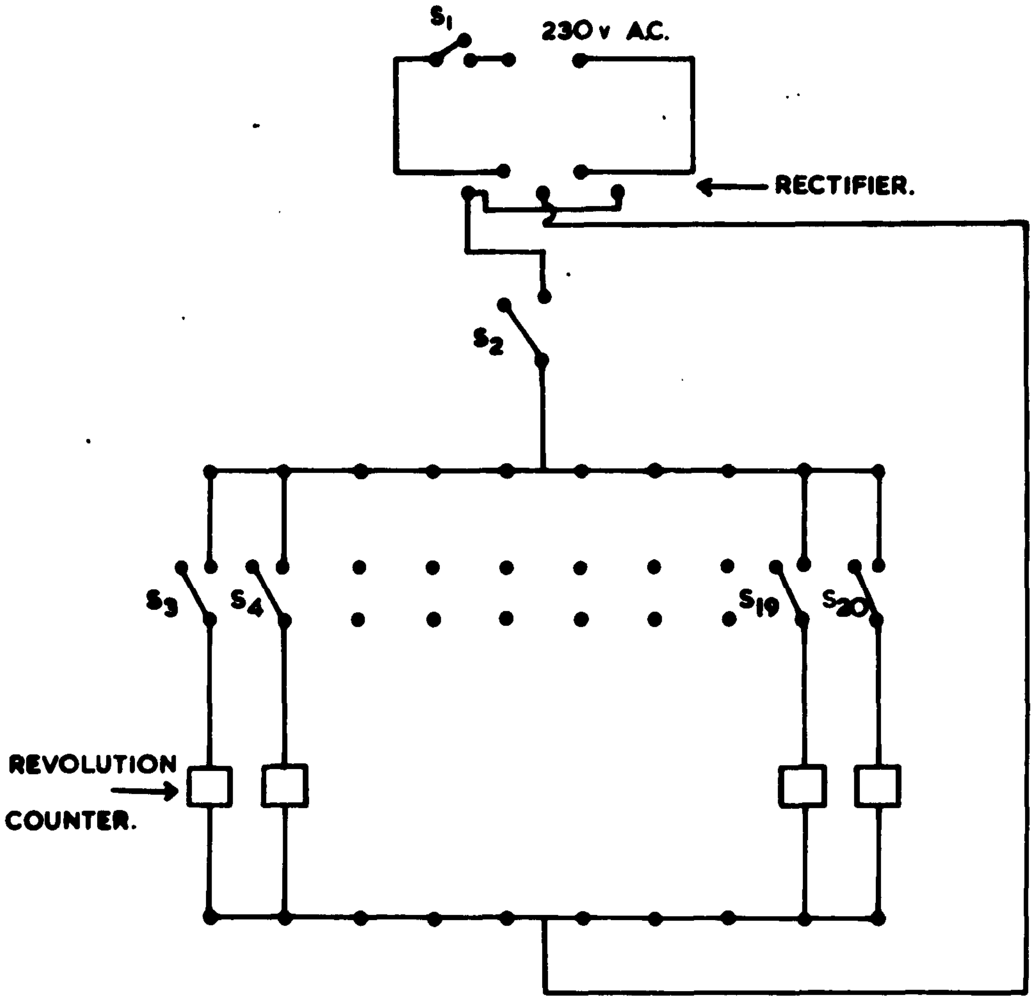
yarn by allowing the lower yarn jaw to fall under gravity, the operation of a knock-off mechanism for break detection is considerably simplified. As is shown in Figure 4, a microswitch arm is placed directly below the centre of the coil assembly and when the yarn breaks the iron rod falls and depresses this arm and opens the appropriate counter circuit (see Figure 11).

The weight of the lower jaw assembly is only 8.5 gms. so the microswitch must be designed to operate reliably on slightly lower loads than this figure. The microswitch BZ-2RW84N15, manufactured by Honeywell Controls Ltd. was the only microswitch to be found satisfactory. This switch requires an operating force of 7 grams weight for it to work reliably over long periods. The 20 microswitches are mounted on two long bars and are located beneath their respective coils. As can be seen from Figure 11, one lead from each microswitch is taken to the appropriate counter and the other leads have a common junction which is an output D.C. terminal of the rectifier. A high stability television rectifier, Automat TV.97, operating on up to 240 v. at a current rating of 200 mA was used.

The counters used are 6 digit resettable Hochschler electromagnetic counters, the British agents for which are Lancashire Dynamo Ltd., Rugeley, Staffs. These counters possess numerous advantages, one being that the space they occupy is a minimum due

FIG. 11.

THE COUNTER CIRCUIT.



to the fact that they are manufactured to be capable of being interlocked to form blocks of counters either vertically or horizontally.

1.6 Automatic recording circuit

Naturally it is preferable if the machine will produce results even if the operator is absent and the design of the machine has incorporated facilities for achieving this.

The uniselector switch, mentioned earlier in 1.3, has 25 positions arranged in a semi-circle which are contacted by a phosphor bronze wiper arm driven from a spindle, on one end of which is a ratchet and pawl to enable steps from one contact to the next to be made. In normal use, the uniselector is moved from one position to the next by the operation of a relay but it was found more convenient to drive the operating spindle directly from a synchronous motor so that the period of rotation of the wiper is accurately known. A small collar with 8 B.A. grub screws serves to link the spindle from the motor with that of the uniselector. A Sangamo Weston synchronous motor making one revolution in two minutes was used. This motor has an operating torque of 850 gram.cms. By switching on this motor at an appropriate time for a set period of time, a record of the instantaneous length and of the tension in each of the 20 yarns can be obtained consecutively.

The method of switching on the motor at the appropriate time was to incorporate a microswitch in the circuit of the synchronous

motor. This microswitch is connected "normally open". Another synchronous motor making one revolution in one hour has an arm fixed to its spindle which is so arranged as to wipe the microswitch extension arm for a period of 2 minutes. The setting of the arm in relation to the microswitch is clearly critical as slight errors mean that the starting point is changed for each successive hourly test. Another arm from the spindle of the one hourly synchronous motor depresses another microswitch to start the paper delivery from the U.V. recorder. This microswitch is connected "normally open" and when closed, the "shot" remote control circuit is shorted and paper is automatically delivered. The amount of paper delivered can be regulated from 0 - 3 metres in steps of 10 cms. Once the shot control has been shorted paper will, however, continue to be delivered in the pre-set lengths until the microswitch arm is released and contact is broken. Therefore it is necessary to have this microswitch on for a very short time before and after the main operation. It is allowable for the microswitch to open just before the main operation ends because if the paper speed has been chosen correctly, sufficient paper will be produced to finish the test.

The whole of this automatic recording equipment is governed by a switch on the main control panel. Should the uniselector wiper be required to move manually only the 8 B.A. grub screw on the motor spindle must be loosened.

1.7 Operating procedure

Having switched in the strain gauge circuit and the inductance bridge circuit, at least 20 minutes should be allowed to let the strain gauges come to a constant temperature. In this time the yarns can be mounted as follows:

1. Rotate the main driving shaft until the position is reached when the electromagnetic clamp has just closed.

2. Set the uniselector switch on the contact number required (say No. 20).

3. Insert the 10 cms. gauge bar into the top clamp.

4. Insert the appropriate lower jaw assembly into the coil No. 20.

5. Affix the lower jaw to the gauge bar.

6. Close the electromagnetic clamps by switch.

7. By observing the galvanometer spot turn the thumb screw under coil No. 20 until balance (or any other required position) is attained. Once set this screw should not need to be adjusted unless a different starting position is desired.

8. Take out the gauge bar.

9. Insert the specimen from the bobbin or other yarn package.

10. Open the electromagnetic clamps.

Position No. 20 is now mounted and the whole procedure is repeated for No. 19, etc.

The initial tensioning of the yarn is possibly open to criticism, the method employed being to clamp the yarn in the lower jaw (the clamps being on), to pass the yarn through the top jaw and to keep the yarn just taut as it comes off the package. The top jaw is then closed. This method, while ensuring that the yarn is not slack between the jaws does not impose a definite fixed initial tension. The tension imposed is estimated to be in the region of 3 grams. Tension in the yarn is developed, however, when the electromagnetic clamps are opened so that the next position can be dealt with. This tension is 8.5 grams assuming that the lower jaw assembly hangs freely. Clearly as the yarns are being mounted, some will have had this weight acting on them for longer periods than others but providing the last yarn to be mounted is given a short time of, say, 2 minutes to be tensioned under the weight of its lower jaw assembly, any effect between stations due to initial tension should be negligible. A tension of 8.5 grams is well below the yield load for the yarns in question and the extension imposed is very small.

1.8 Some initial problems

It was found that as the electromagnetic clamps opened and the lower jaw assembly took up a new position, the yarn was subject to an impact force of approximately 15 - 20 grams. This force was not considered excessive but steps were taken to reduce it by damping.

The glass tube in which the soft iron rod is free to slide was sealed off and half-filled with various liquids. Liquid paraffin and glycerine were both tried, the latter proving more successful due to its higher viscosity (8.5 as against 0.02 c.g.s. units). For a liquid of still greater viscosity, a mixture of benzene and polystyrene could be used. It was felt, however, that any advantage to be gained from damping would be offset by the delicate work needed to readjust the break-detecting microswitches. This readjustment would entail fitting a fine spring to balance the weight of the glass tube and liquid. The tube would have to rest on the microswitch arm all the time without depressing the switch itself. In the initial tests conducted, this improvement has not been found necessary but if future results suggest its value, it could be incorporated.

Another previously unforeseen disadvantage of the clamping system is the liveliness and potential torsional energy of a twisted yarn as it is subjected to repeated oscillations. When the clamp is opened the yarn has a tendency to untwist; to counteract this a needle which slides between two upright guides is attached to the lower jaw assembly at right angles to the yarn and also at right angles to the line of clamping. This restrains the yarn's motion to approximately 1/10th of a revolution, which can be neglected.

CHAPTER II

Results and Discussion

2.1 Wave form of the imposed cycle

The wave form of the imposed oscillation given to the top jaw is as shown in Figure 12 and as can be seen approximates to a sine wave very closely. The curve was obtained by having a lower jaw assembly attached rigidly to the corresponding top jaw, thereby letting the motion of the top jaw be represented by the movement of the soft iron rod in the coil assembly. As the motion is confined to a stroke of 0.24 cms. amplitude the portion of the curve just above the balance point was used and as this portion is linear, the curve produced is a good representation of the movement of the top jaw. The recorded wave is of course an A.C. signal. If the curve is closely scrutinized there are slight departures from the true sine wave; these discrepancies in the wave form have been theoretically dealt with in 1.1 and in practice similar results have been found to those predicted.

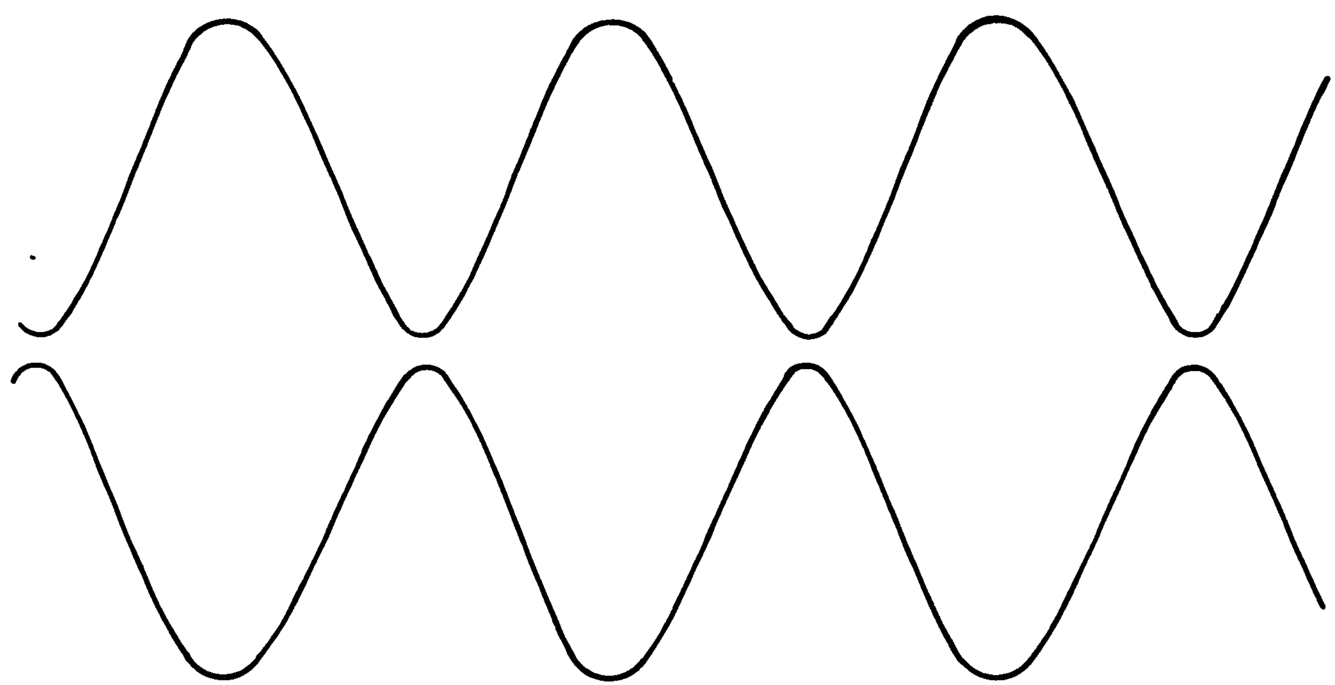
2.2 Results

2.2.1 Typical traces

A trace from the U.V. recorder is reproduced in Figure 13. As can be seen, three galvanometers are being used, one for length measurements (A.C. trace 50 c/s), one for tension measurements

FIG. 12.

WAVE FORM OF THE STROKE IMPOSED UPON THE YARN.

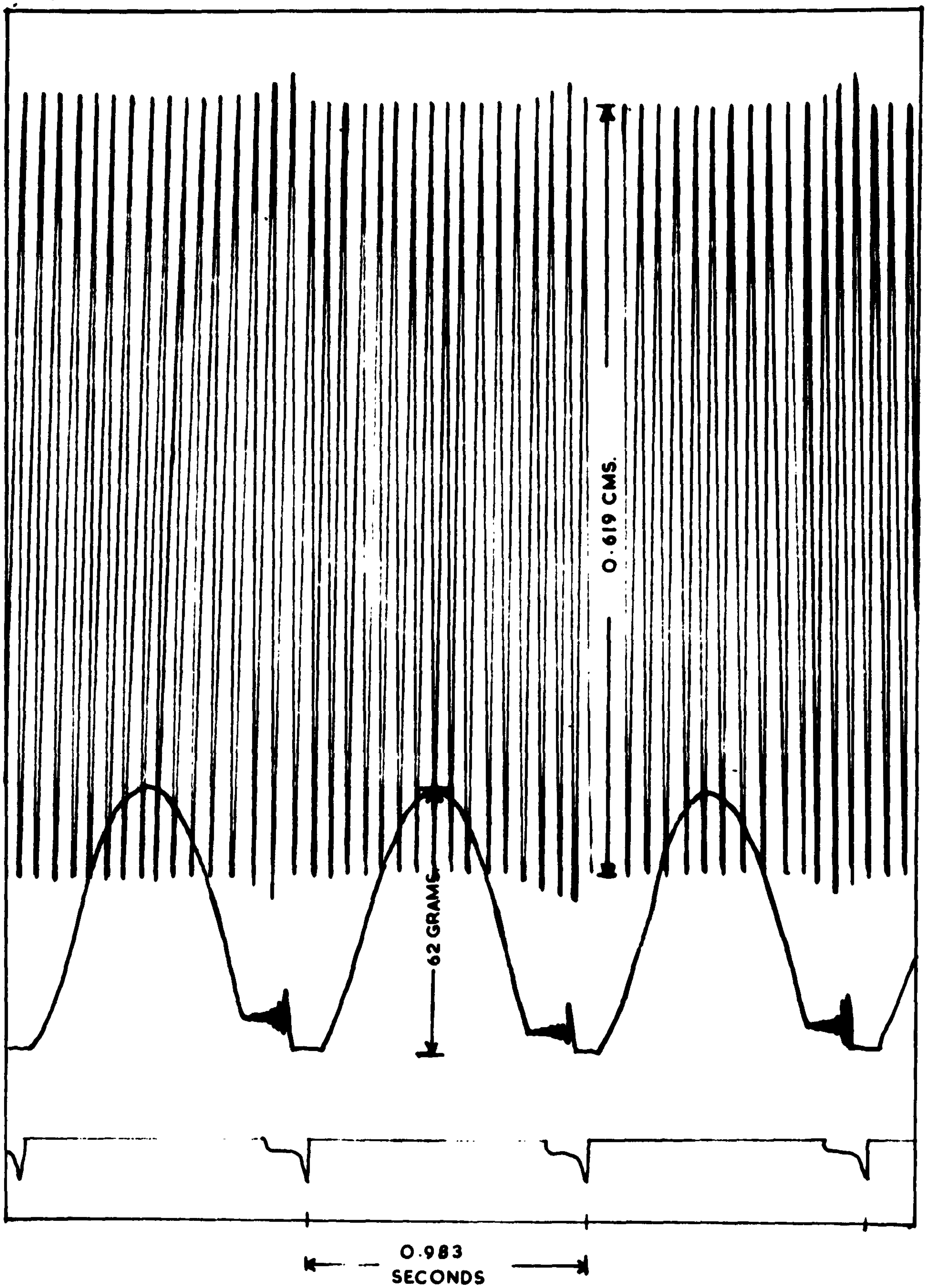


2.4 % STROKE.

61 CYCLES per MINUTE.

FIG. 13.

A TYPICAL RECORD CHART.



(modified D.C. sine wave) and a third as a timing trace. The frequency of oscillation on this trace is 61 cycles per minute and for ease of reproduction the A.C. signal has been reduced on the diagram to one third of its actual frequency.

When the electromagnetic clamps open, indicated by the sharp fall in the timing trace, it can be seen that there is a short time lag (approximately 70 milliseconds) before the soft iron rod falls to its new position. This is also shown up by the impact tension on the yarn at this point which amounts to approximately 15 grams. The tension on the yarn then drops to 8 grams, the weight of the lower jaw assembly. The electromagnetic clamps then close and there is another short time lag before the tension of the yarn begins to increase on the upstroke of the oscillating movement. The two time lags are caused by the finite length of time required for the hard rubber jaws of the electromagnetic clamp to contact and grip the brass section of the lower jaw assembly.

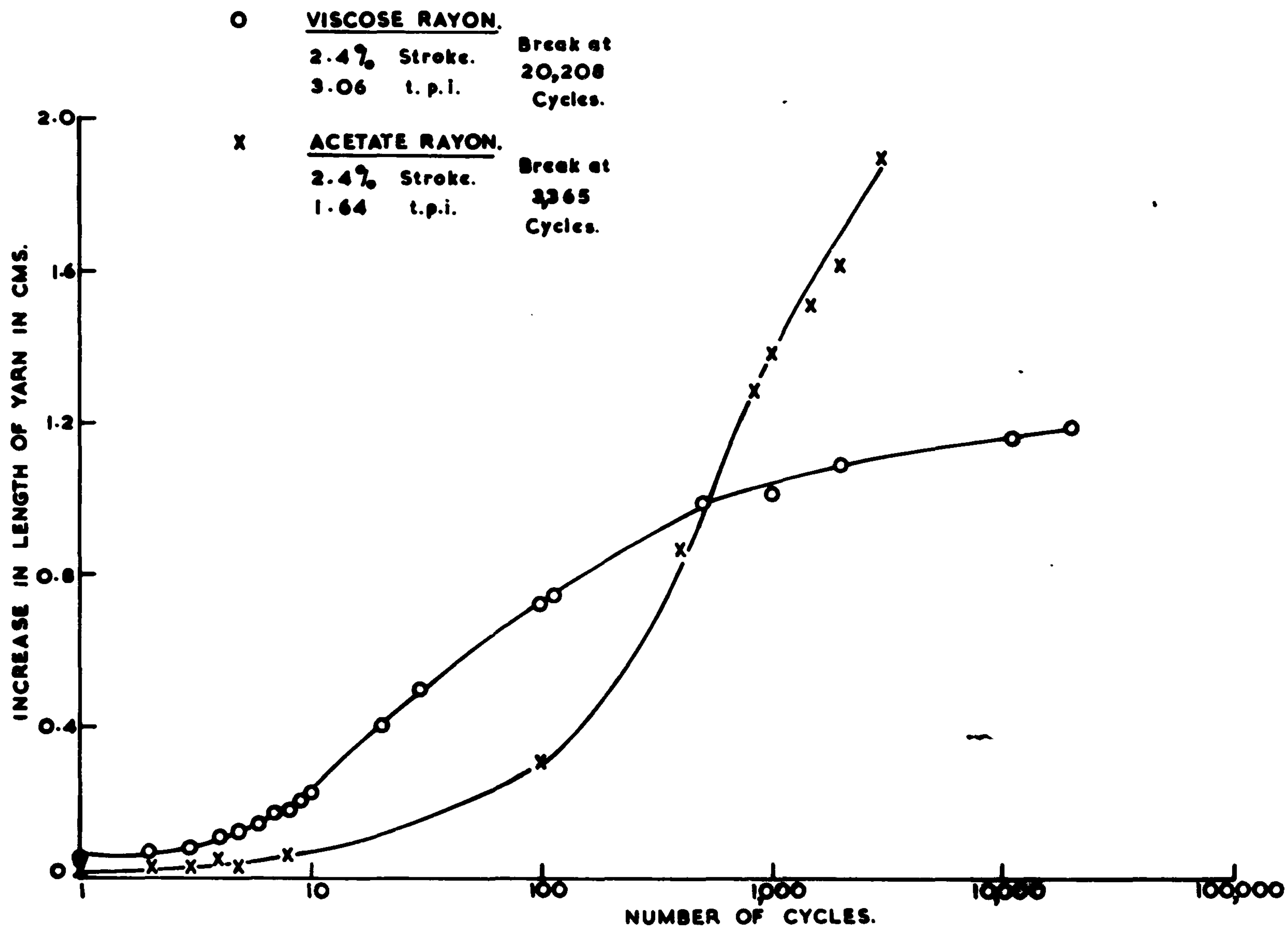
The amplitude of the A.C. signal does not alter while the yarn is being extended; if this were not the case slippage in the clamp would be prevalent and this fault would have to be rectified immediately.

2.2.2 Results and discussion

Figure 14 shows the growth in length during cycling of a viscose and an acetate sample. Both yarns are of very low twist

FIG. 14.

GROWTH OF YARN UP TO THE BREAKING POINT.



factor and both were tested at the same imposed stroke length of 2.4%. Both curves show a sigmoidal shape up to the breaking point.

For the viscose yarn, the greater part of the increase in length occurs between 10 and 1000 cycles whereas the acetate yarn begins to increase rapidly only after the first 100 cycles. If the curves had been depicted on a scale graph rather than on log paper the sigmoids would be a lot flatter in both cases.

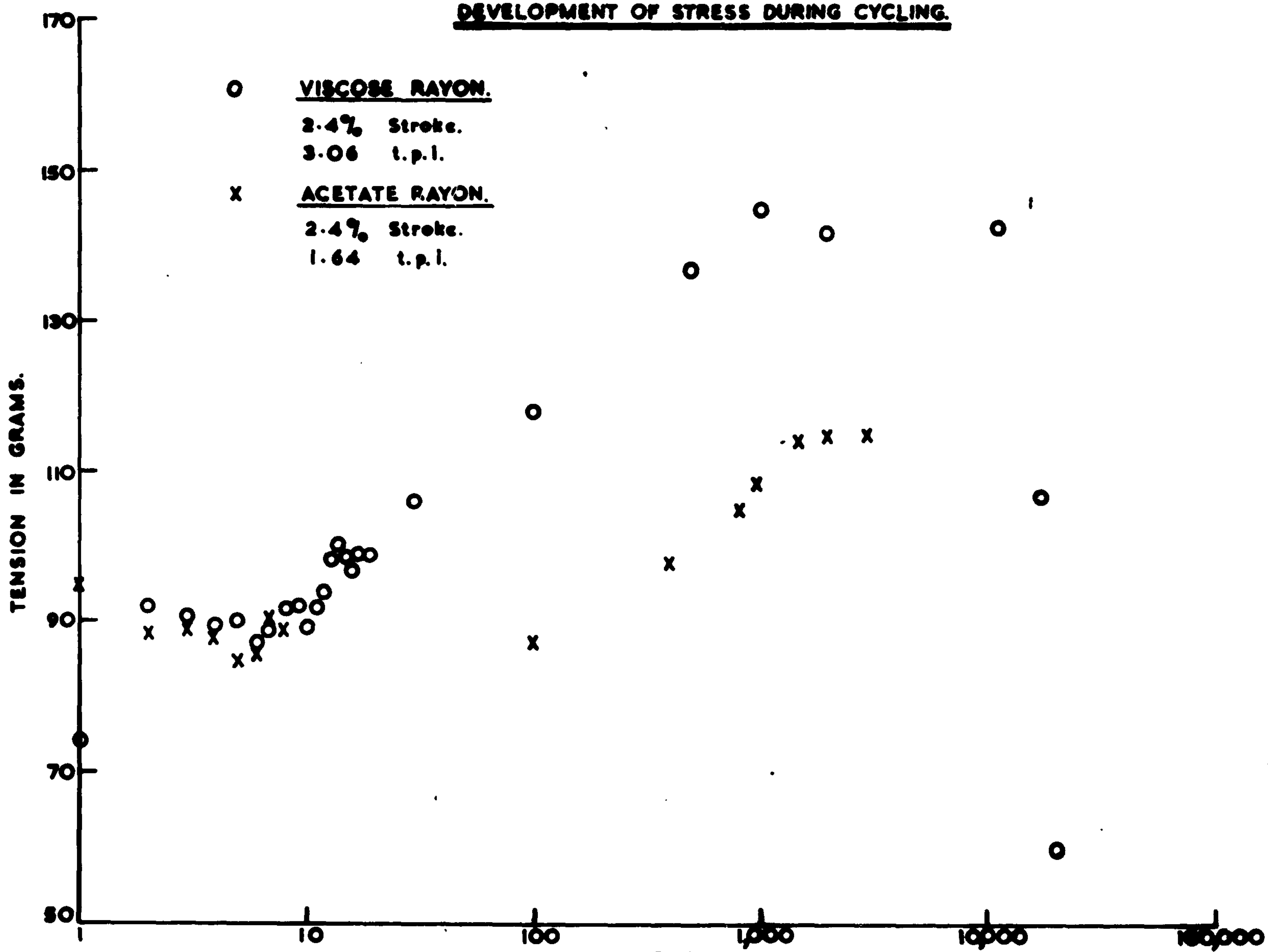
Figure 15 shows the development of the stress in the yarn as it is continuously cycled. In both yarns an increase in the tension is developed, this being more noticeable in the case of the viscose yarn. During the first 10 cycles there is considerable scatter in the results for peak tension and this is difficult to account for: one possible explanation is that the yarn still possesses good elastic properties and in the short space of time when the clamps are opened, a certain amount of stress relaxation takes place.

The number of cycles to break for the viscose yarn was found to be 20,208 and for the acetate yarn 3365 cycles. Five other tests were conducted with acetate yarns and the number of cycles to break was 2819, 1547, 2015, 2614 and 3466 cycles respectively. Considering the many variables associated with a fatigue test, this variability in fatigue life was considered acceptable.

Further tests are in progress and the results will be available in the first quarterly report (December, 1962).

FIG. 15.

DEVELOPMENT OF STRESS DURING CYCLING.



PART II

GEOMETRIC STRUCTURE AND FORM OF YARNS

CHAPTER III

Apparatus for Study of Rubber Models

3.1 Introduction

Previous studies of the untwisting of yarns, coated on the outside with a coloured paste, have indicated that yarn may be twisted like a flat ribbon. To aid in the interpretation of yarn structure, some large-scale models were made and described in the last Annual Report (reference on page 111). The most interesting of these were rubber strips, and a more extensive study of the twisting of flat strips of rubber has been carried out in the current year.

At first, models were made by twisting rubber strips held between the two jaws of a twist tester. The jaws of the twist tester were fixed in position and thus the models were made under the condition of constant length. Three different ribbon widths (1 cm., 0.7 cm. and 0.4 cm.) were used. Two distinct forms of structure were observed: a twisted form appearing at low twists, and a wrapped form appearing only at high twists. In the last

annual report of this work it was mentioned that the wider strips start wrapping at a lower number of turns per unit length than the narrower strips. Very little work was done at that stage, but, it seemed likely that there must be a certain amount of minimum twist for different denier yarns before it starts getting a wrapped structure as shown in Figure 73B of the last report. Later on models were made out of various widths of ribbon and also at constant tension to investigate in greater detail into this sort of enlarged structure.

3.2 Twisting of rubber strips

Observations were made on 2.5 mm., 4 mm., 5 mm., 6 mm., 7.5 mm., 8 mm., and 1 cm. wide rubber strips. The strips were twisted under two different conditions:

(i) Twisting at constant length - by gripping the rubber strips in the two jaws of a twist tester and twisting them, the two jaws being screwed in position.

(ii) Twisting at constant tension - by gripping one end of the rubber strip in the rotating jaw of a twisting head, while the other end was gripped in the jaw of a perspex trolley sliding on rails, so that contraction in length could take place as twist was introduced in the strip. A tensioning weight could be hung from the other end of the trolley. The assembly of twisting head and trolley on rails was specially made for this purpose.

Four different types of observations were made:

(a) a study of the distribution of twist in the strip, specially after wrapping has started. For this purpose the total number of turns introduced, the number of turns at which wrapping occurs, the length of strip wrapped, and the number of turns in the wrapped part were observed. In the case of twisting under constant tension the position of the trolley along the scale was observed. This would give any contraction in length taking place during the introduction of twist.

(b) a study of the effect of twist on the helix angle. For this purpose the helix angle was measured after each turn of twist was introduced, and when the wrapping had started the helix angle was measured in both twisted and wrapped part.

(c) a study of the effect of length of strip on the turns per unit length at which wrapping occurs. Different lengths of each particular width of rubber strip were twisted and the exact number of turns at which jamming occurred noted. The lengths of strip used were 25 cms., 20 cms., 15 cms., 10 cms., and 5 cms.

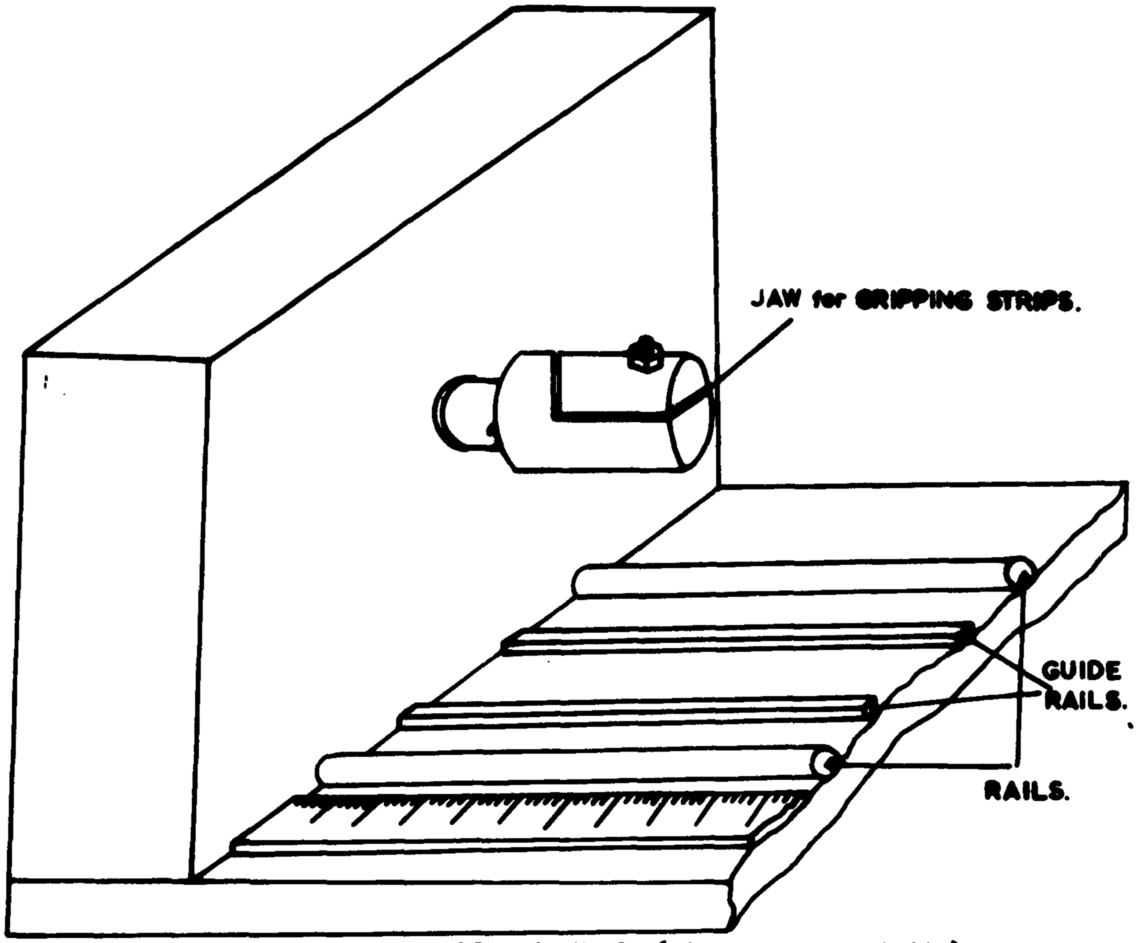
(d) a study of the effect of twisting tension on the helix angle and turns per unit length at which wrapping occurs. The strips were twisted under tensions of 2.5 gms., 5.0 gms., 7.5 gms., 10 gms., 12.5 gms. and 15 gms.

Description of rubber twisting apparatus

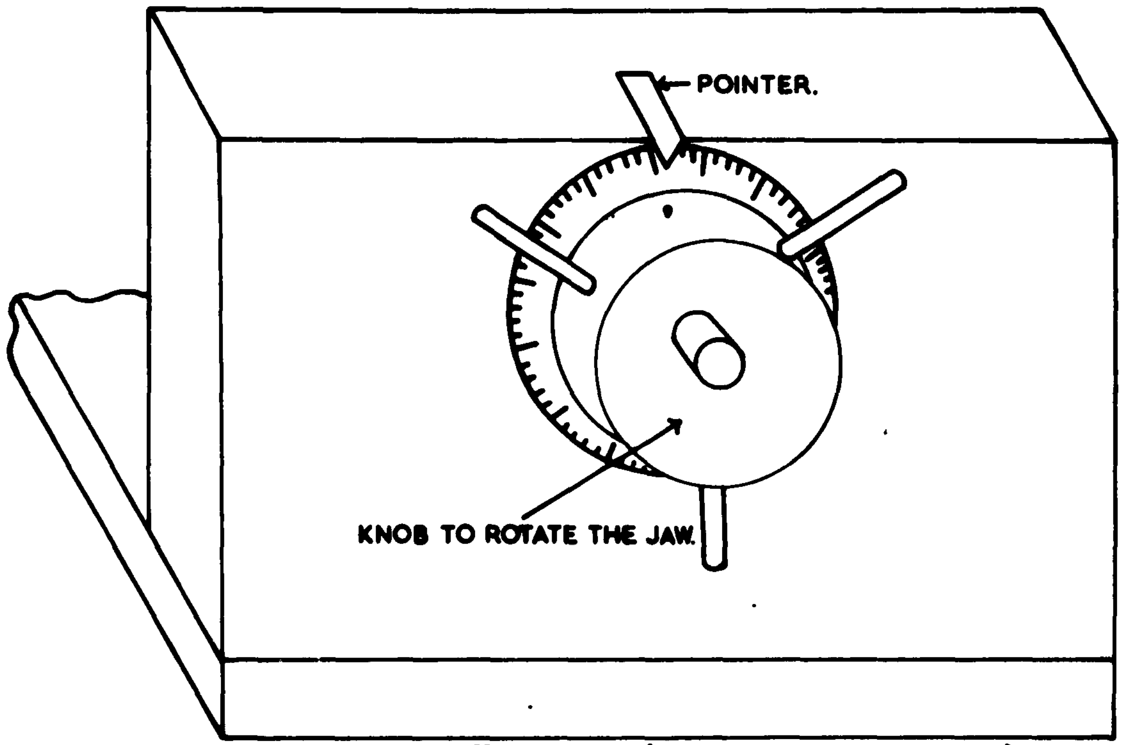
The apparatus for twisting rubber strips at constant tension consists mainly of a trolley, movable on a pair of rails and a twisting head. The rails were made out of 1/4" polished mild steel rods and were mounted on a wooden base. A metre scale was screwed on to this wooden base parallel to the length of the rails. On one end of this wooden base was fixed a vertical wooden block. A hole was drilled into this wooden block and a brass collar was pushed into this hole. Through this collar was passed a spindle, one end of which was made into a jaw (similar to the jaw of a twist tester, as shown in Figure 3.1(a)), and on the other end (away from the rail side) was attached a handle for imparting rotational movement to the jaw. A circular scale was attached to this handle and was mounted on the spindle so that the number of full rotations as well as fractions of rotation given to the jaw could be measured. A pointer fixed on the wooden block would read the exact number of rotations given to the jaw. Figure 3.1(b)

To reduce the frictional effect of the trolley, it is very desirable to have the trolley as light as possible and the wheels attached to it as frictionless as it could be. For this purpose the trolley was made out of 1/8" and 1/16" thick perspex sheets. The wheels used have ball bearings and were found to be quite suitable for our purpose. A hook also made of perspex was attached to

FIG. 3.1.



TWISTING HEAD. (View from the rail side.)



TWISTING HEAD. (View away from the rail side.)

the other end of the trolley to which a string could be attached. The string was passed over a frictionless pulley at the other end of the wooden base and weights could be hung freely on this end of the string. Thus by hanging different weights, varied tensions for twisting the rubber strips could be achieved. A pointer was attached to the trolley to give its position on the scale attached on the wooden base board, enabling contraction to be measured.

In order to keep the helix angle measuring instrument always parallel to the length of the strip, guide rails were mounted on the wooden base board. These guide rails were made out of wooden strips and were fixed between the rails and parallel to it as shown in Figure 3.1(a)

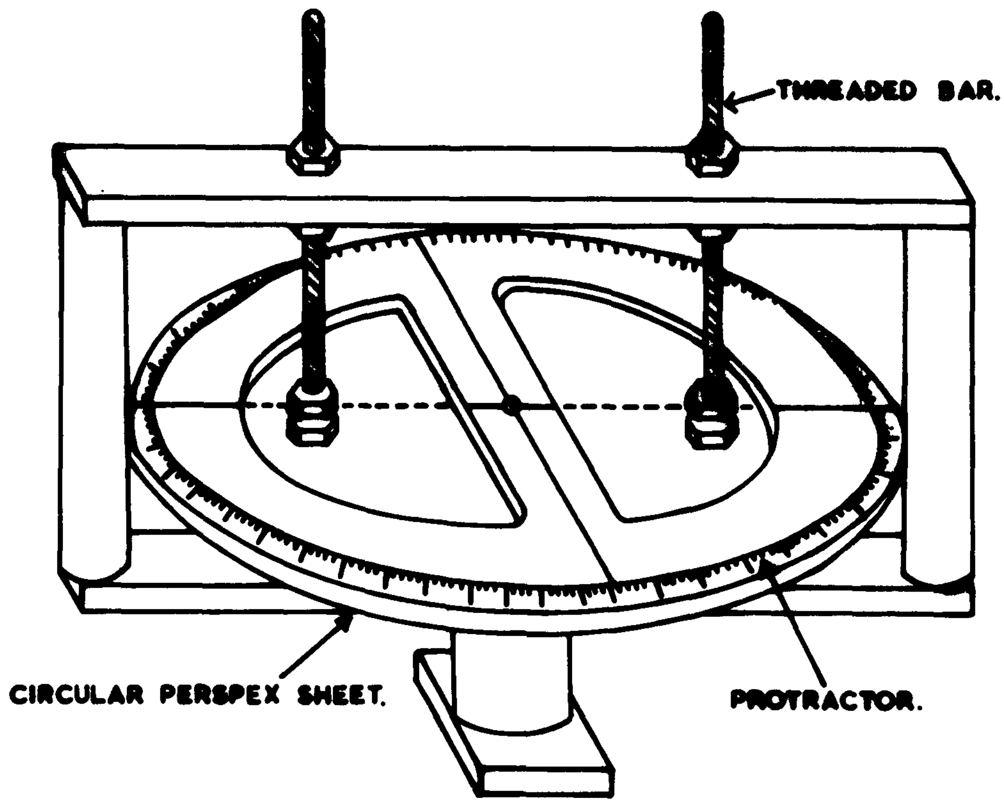
For measuring the helix angle a circular perspex protractor was mounted on another circular transparent perspex sheet by means of a screw. This screw was tightened at the centre of the protractor and the perspex sheet. Thus a relative rotational movement could be given to the protractor on the circular perspex sheet. Two diametrical lines perpendicular to each other were marked on the circular perspex sheet. This assembly of the protractor and circular perspex sheet was mounted on a special type of stand so that the diametrical line on the perspex sheet could be brought on top of the rubber strip (held between the rotating jaw of the twisting device and the jaw on the trolley), and made parallel to the axis of rotation of the strip. The stand

for mounting the protractor consists of a rectangular frame as shown in Figure 3.2. The frame was made out of two vertical circular perspex rods and two rectangular horizontal strips. The frame itself in its turn was mounted on a vertical bar, the base of which was made into a rectangular block of the same size as to fit into the groove of the guide rails. Thus the whole assembly of the protractor could be slid along the length of the rubber strip and helix angle measured at different positions. The protractor and the circular perspex sheet unit were mounted on the frame by means of two threaded bars from the top horizontal strip of the frame. Nuts were used in pairs to adjust the height of protractor from the bottom rectangular strip. The rubber strip was passed between the protractor and the bottom strip of the frame. While measuring the helix angle for thin rubber strips the protractor could be brought down nearer to the strip to measure the helix angle accurately. Slots were made on the top horizontal bar of the frame to observe the readings accurately.

For measuring the helix angle the 0^{o}-180^{o} line on the protractor was made to coincide with one of the diametrical lines on the perspex sheet and the other diametrical line perpendicular to the previous one was set parallel to the axis of the rubber strip before twist was imparted. After the twist was introduced, the 0^{o}-180^{o} line on the protractor was brought to coincide with

FIG. 3.2.

HELIX ANGLE MEASURING INSTRUMENT.



the twisted edge of the strip. The angle between this diametrical line on circular perspex sheet and the 0^{o}-180^{o} line on the protractor gives the helix angle, and could directly be measured.

A magnifying glass was used over the instrument to measure the helix angle accurately.

3.3 Measurement of strip properties

Torsional Modulus - The statical method of applying a couple to the rubber strip by means of a mass supported by a string wound round a wheel was used. The apparatus was specially designed for this purpose to suit the material like rubber, and is explained below:

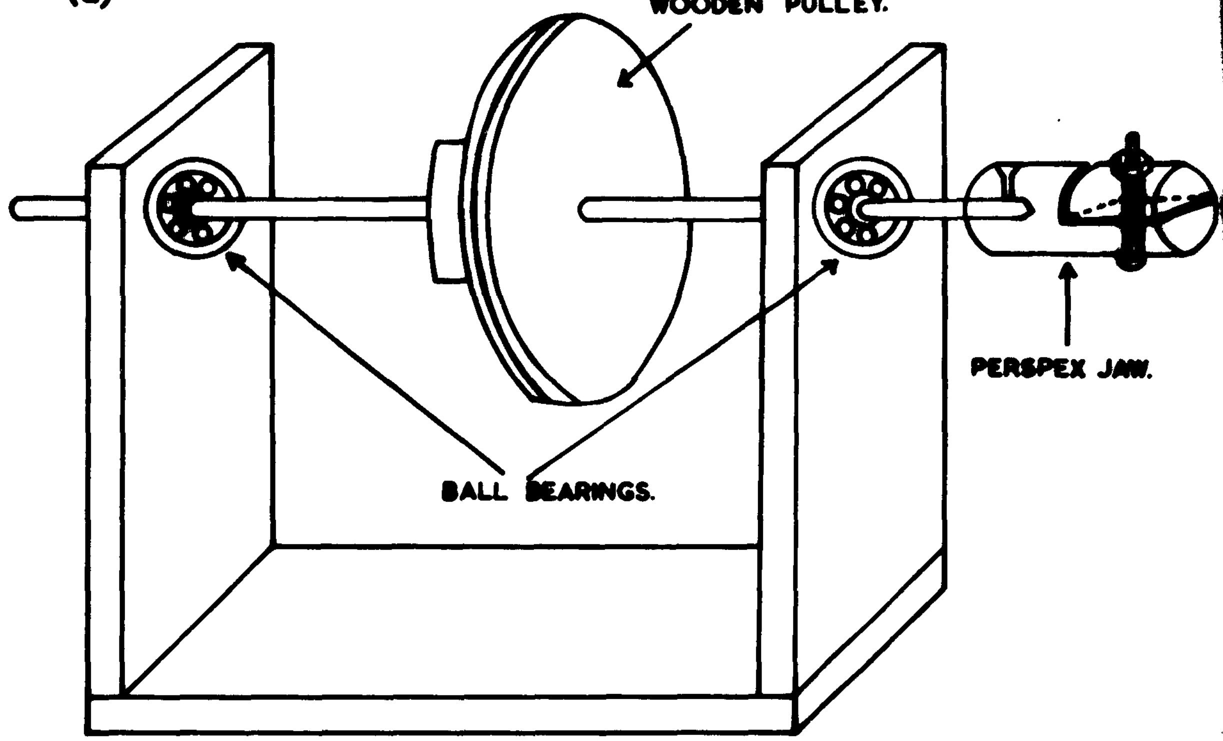
Apparatus for measuring torsional modulus. - A jaw for gripping the rubber strip was made out of perspex and was mounted on one end of a 1/8th inch silver steel rod. This steel rod was then passed through two ball bearings which in their turn were mounted on a bracket as shown in the Figure 3.3. A pulley made of thin plywood was slipped on to the steel rod and screwed in position as shown in Figure 3.3(a). Light materials like perspex, plywood and 1/8th inch steel rod were used to keep the weight of the twisting unit as small as possible. To avoid any frictional force acting on the spindle of the twisting unit, during the rotation of the jaw, the spindle was mounted on ball bearings.

The whole twisting unit mentioned above was mounted on one end of a long wooden board which had a pair of rails running along the

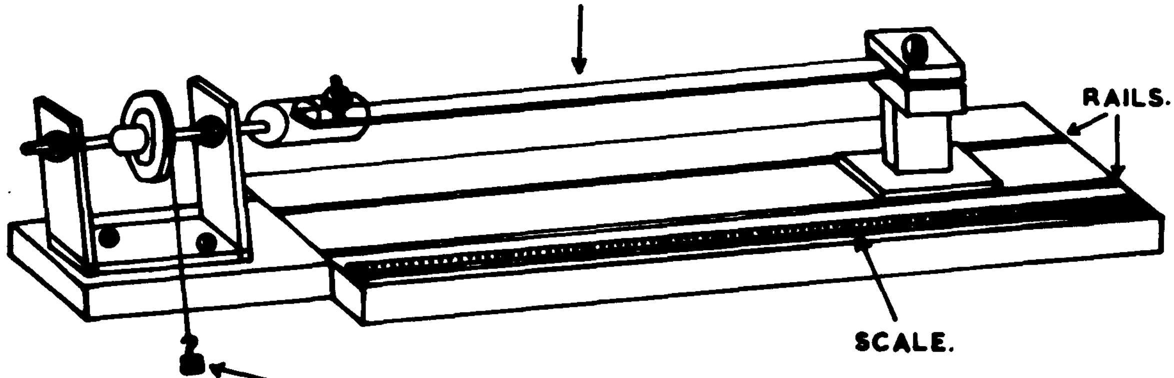
FIG. 3.3.

APPARATUS FOR DETERMINING TORSIONAL MODULUS.

(a)



(b)



length of the board. A clamp was mounted on those rails which could be slid on these rails and could be screwed at any position to have varying length of specimen between the twisting jaw and the clamp. A wooden scale was screwed parallel to the rails, as shown in Figure 33(b), to measure the length of specimen at any time during the experiment.

Determination of the torsional modulus. - The rubber strip is clamped between the two jaws of the twisting unit such that there is no twist in the rubber strip. When a mass M gms. is fastened on to the string, and it is allowed to drop, the wheel will revolve till the couple due to the elasticity of the rubber strip balances the couple due to the mass M. If the radius of the wheel be R^1 cms., this couple is MgR^1 dyne-cms., where g cm. per sec^2 is the acceleration due to gravity.

If the length of the strip be l_1 cms. and n_1 be the number of turns introduced into it before the internal couple balances the applied, then from equation

$$\frac{2}{3}\pi\eta\, bd^3 \frac{n_1}{\ell_1} = MgR^1 \qquad \text{.............. (3.1)}$$

$$\text{or } \eta = \frac{3}{2} MgR^1 \frac{\ell_1}{n_1} \cdot \frac{1}{\pi bd^3} \qquad \text{............. (3.2)}$$

Thus measuring ℓ_1, n_1, b, d, and R^1 and knowing the value of M, the torsional modulus η can be calculated.

Bending Modulus

To determine the bending modulus of rubber strip, the principle of determining the bending length of fabrics for estimating their floxural rigidity was used.

Apparatus for measuring bending length

A fixed angle flexometer[22] was used for this purpose. The essential features of the fixed-angle flexometer are shown in Figure (Refer J.T.I.,1959, P772)

On the horizontal platform P rests a slide S graduated in bending length in cm. When the front edge of the slide coincided with the front edge of P, the zero the scale on S coincides with a datum line D on the instrument. Two sighting lines L_1 and L_2 passing through the upper forward edge of P and inclined at an angle of $41\frac{1}{2}^{\circ}$ below the horizontal, are inscribed on the transparent side pieces of the instrument. The under surface of S is covered with rubber and the upper surface of P is polished so that, when S is moved, it will carry forward a specimen placed between the slide and the surface P. The weight of the slide should be sufficient to keep the specimen flat and in close contact with the upper surface of P.

Determination of bending modulus

A rectangular strip of the specimen is supported on a horizontal platform in a direction perpendicular to one edge of the

platform. The strip is traversed in the direction of its length so that an increasing part overhangs and bends down under its own weight. The length of the overhanging part of the specimen is noted when the tip of the specimen has reached a plane passing through the edge of the platform and inclined at an angle $41\frac{1}{2}^{o}$ below the horizontal.

The bending length is obtained directly from this observation. The bending modulus is calculated from the formula given in the British Standards Handbook[23] and is explained below.

If ψ be the Flexural Rigidity and q be the bending modulus then

$$\psi = wc^3 \quad \text{gm.cm.}$$

where w is the weight of the specimen in grammes per square centimetre and c is the bending length in cms.

and the Bending Modulus $q = \dfrac{12\psi}{d^3}$ gm./cm^2 .... (3.3)

where d is the thickness of the material in cms.

<u>Comparison of bending modulus with Young's modulus</u>

The Young's modulus of the specimen was also measured on the Instron machine to compare the bending modulus of the material (rubber) with its Young's modulus and see whether the Young's modulus values could be used for calculations of energies, etc. in place of bending modulus.

It was observed that there was very little difference in the values of bending modulus, calculated from the bending length method,

from the Young's modulus, determined on Instron, and hence for practical purposes the values of Young's modulus have been used.

Measurement of Thickness of Ribbon

For measuring the thickness of the ribbon Reynolds and Bransons Thickness Tester was used.

The sample is pressed between two circular anvils, each having a surface area of 1 $cm.^2$, one of which is fitted to a lever mounted on a flat spring which acts as frictionless bearing. The lever has a recess in which the 5 gram. pan is suspended. Accordingly, a pressure on the sample of 5 gms. per $cm.^2$ is obtained. By adding small weights this may be increased. The dial gauge is provided with metric scale (readings to 0.01 mm.).

After the sample has been put in between the two anvils, the gauge spindle is advanced slowly by means of the large knurled screw, until the red indicator lights up brightly. The reading is noted which directly gives the thickness of the sample.

This instrument has been used for the measurement of the thickness of small samples of fabric under pressures from 5 gms./cm^2 upward, as described in British Standards Handbook No. 11, pages 119-120.

CHAPTER 4

Results on Twisting of Rubber Materials

4.1 General

The observations made during the process of twisting rubber strips have been given in Tables 4.1 to 4.11. Out of these, Tables 4.1 to 4.4 show the observations made for twisting strips at constant length and Tables 4.5 to 4.11 are the observations for twisting under constant length.

4.2 Twisting at constant length

From the graph (Fig. 4.1) showing the distribution of length of strip in the twisted and the wrapped part it can be seen that wrapping does not occur until certain minimum number of turns have been introduced into the strips. This minimum number of turns at which wrapping would occur depends upon the width of the strip, and is more for narrower strips than the wider ones. Although the observations for three ribbon widths have been plotted on the graph (Fig. 4.1). it can be seen from the tables that strips of all wdiths follow the same trend. Once the wrapping has started, the length of strip in the twisted part goes on decreasing as a greater number of turns are introduced in it, whereas the length in the wrapped part goes on increasing until nearly all the length has taken the wrapped form.

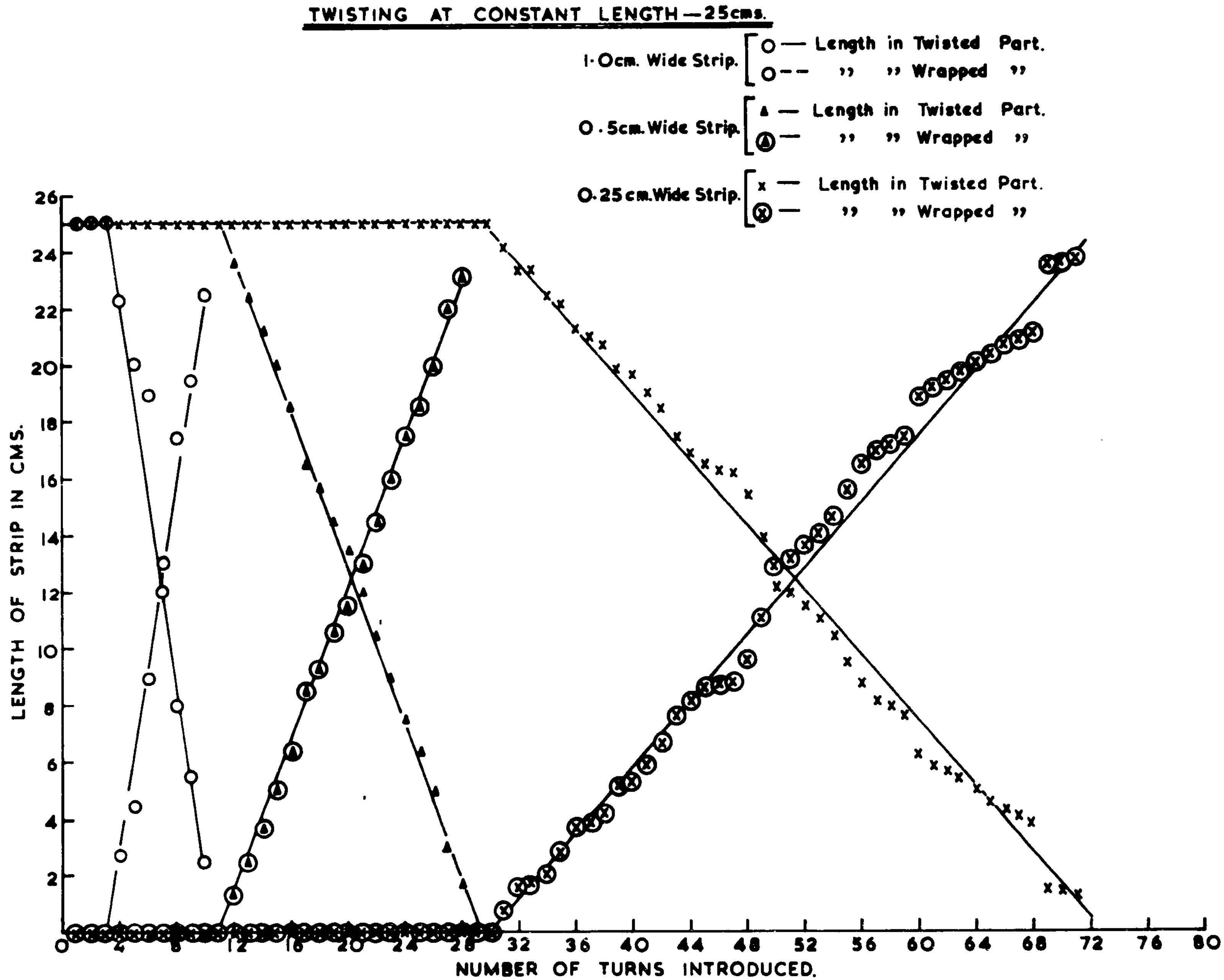

TWISTING AT CONSTANT LENGTH—25cms.
1·0cm. Wide Strip.
Length in Twisted Part.
" " Wrapped "
O·5cm. Wide Strip.
Length in Twisted Part.
" " Wrapped "
O·25cm. Wide Strip.
Length in Twisted Part.
" " Wrapped "
LENGTH OF STRIP IN CMS.
NUMBER OF TURNS INTRODUCED.
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
32
36
40
44
48
52
56
60
64
68
72
76
80

FIG. 4. 1.

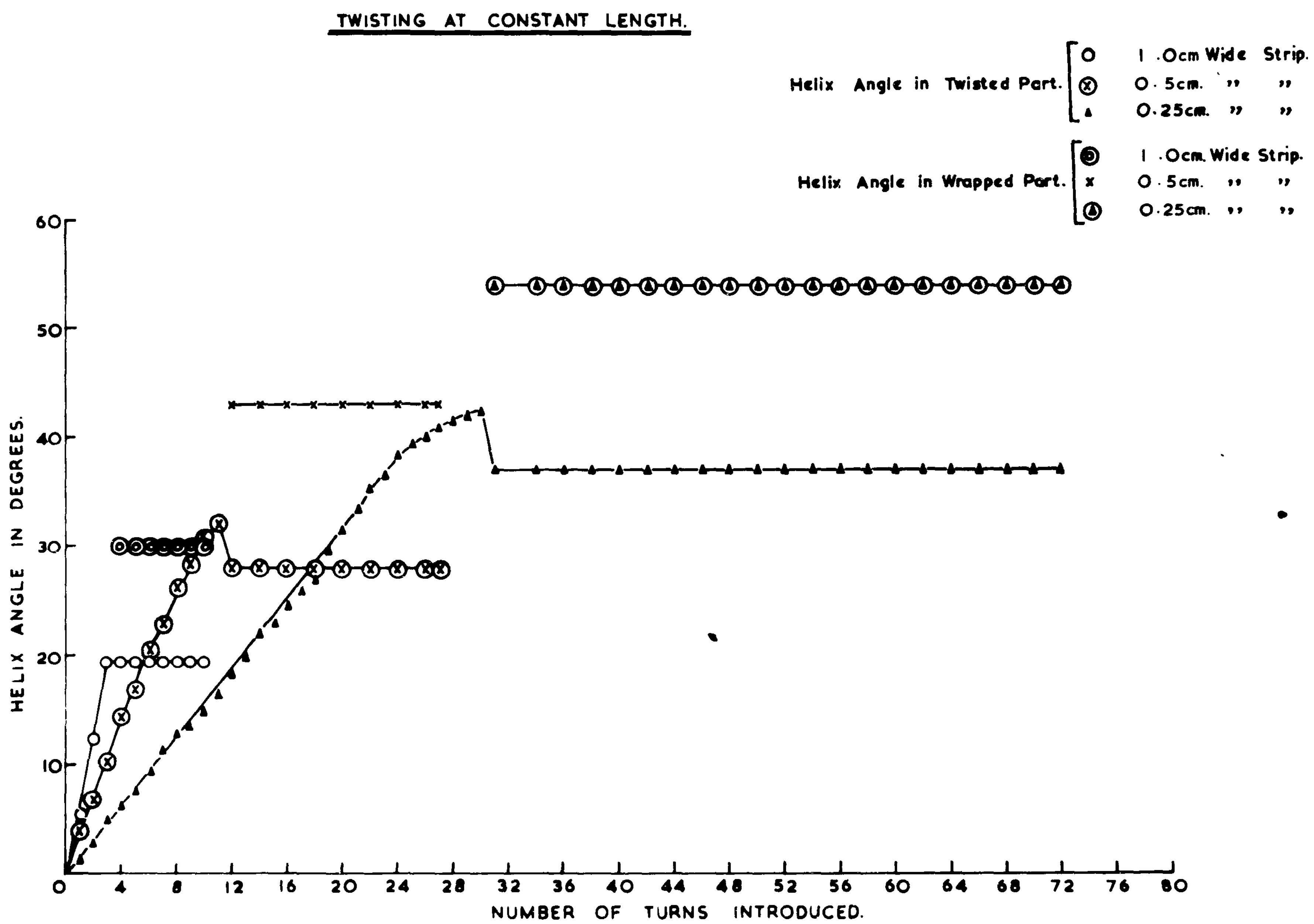
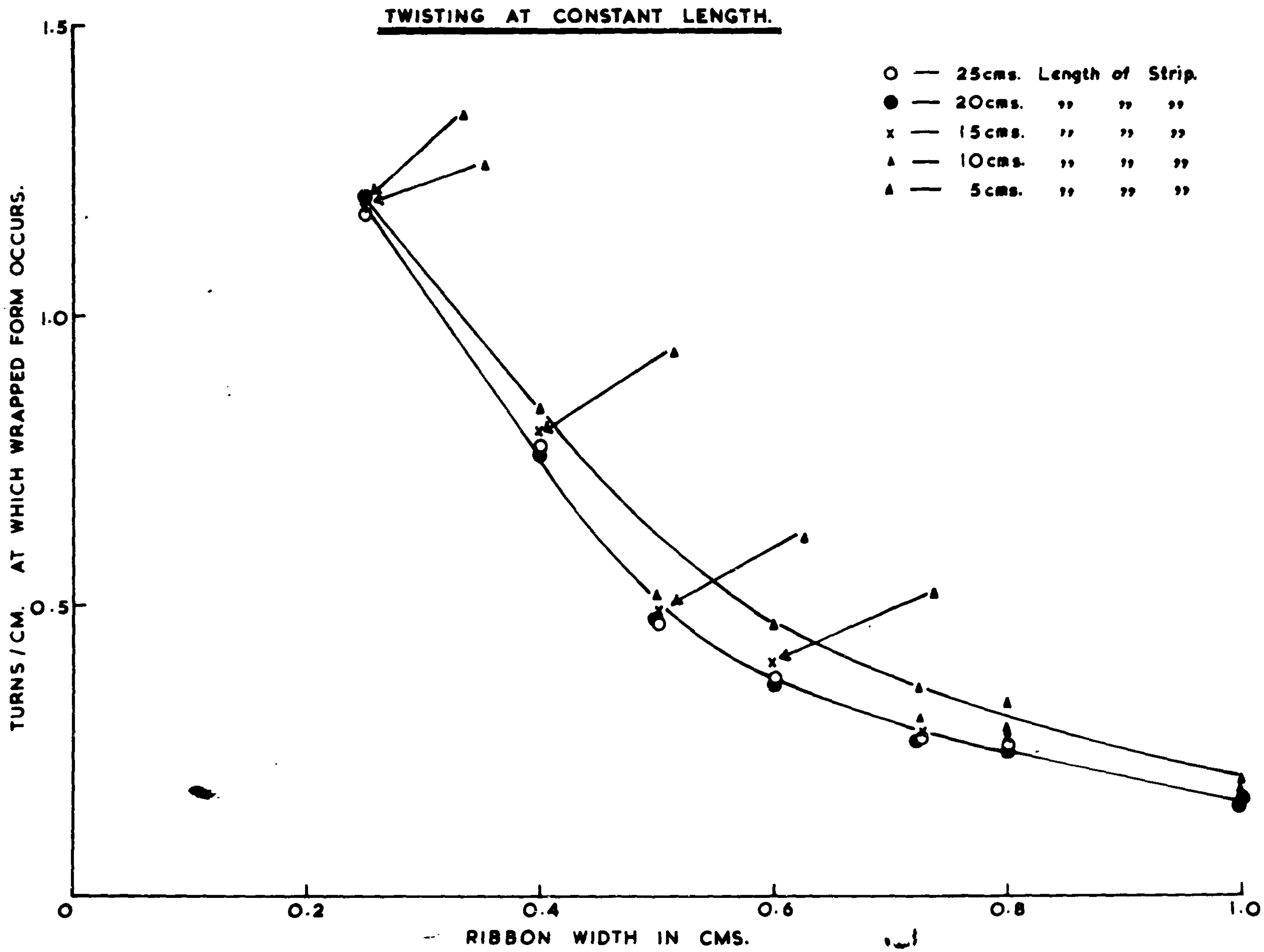

TWISTING AT CONSTANT LENGTH.
Helix Angle in Twisted Part.
1.0cm Wide Strip.
0.5cm. " "
0.25cm. " "
Helix Angle in Wrapped Part.
1.0cm. Wide Strip.
0.5cm. " "
0.25cm. " "
HELIX ANGLE IN DEGREES.
NUMBER OF TURNS INTRODUCED.

FIG. 4.2.

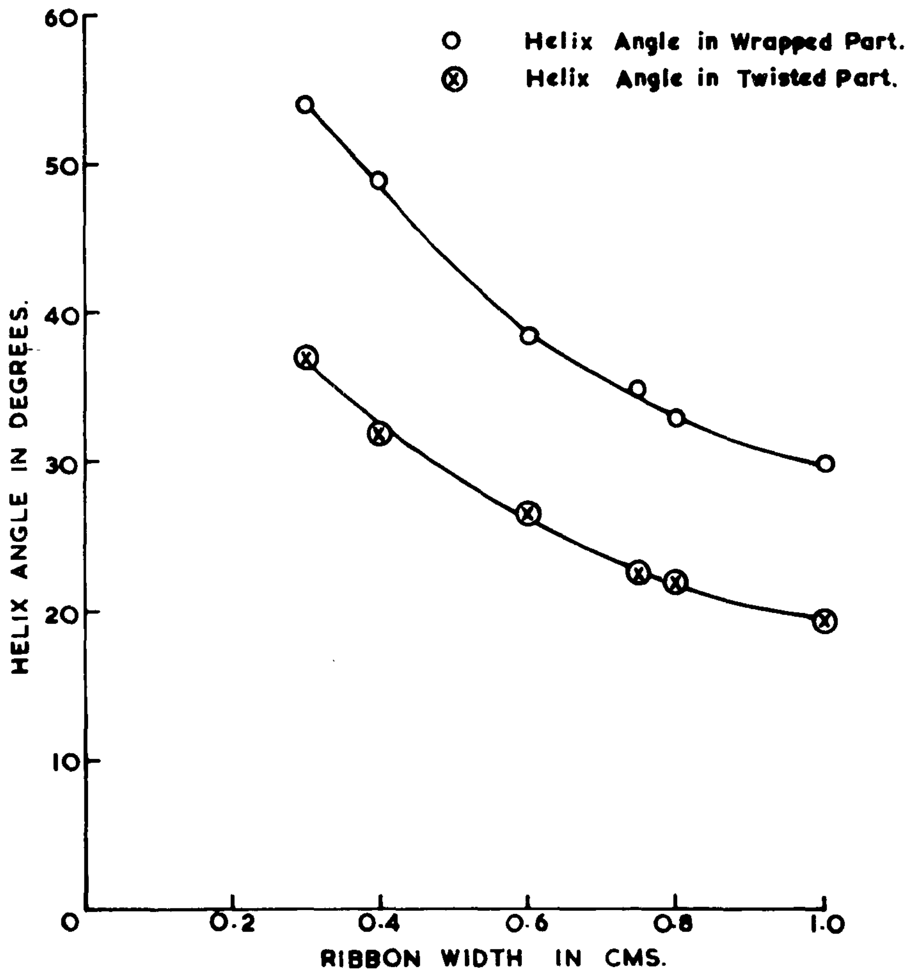


TWISTING AT CONSTANT LENGTH.
O — 25cms. Length of Strip.
● — 20cms. " " "
x — 15cms. " " "
▲ — 10cms. " " "
▲ — 5cms. " " "
TURNS / CM. AT WHICH WRAPPED FORM OCCURS.
1.5
1.0
0.5
0
0.2
0.4
0.6
0.8
1.0
RIBBON WIDTH IN CMS.

FIG. 4.3.

FIG. 4. 4.

TWISTING AT CONSTANT LENGTH.



It can also be observed that some portion of the length cannot be wrapped because of the two ends of the strip being gripped in the jaws.

The helix angle in the twisted and wrapped part were measured and Fig. 4.2 shows that helix angle in the twisted part goes on increasing until wrapping occurs. After wrapping has started, the helix angle in the twisted part and the helix angle in the wrapped part remains constant. Once the wrapped form has been obtained any fresh number of turns introduced into the strip goes into wrapping fresh part out of the twisted part and thus the helix angle in the twisted and wrapped part are maintained constant.

Graphs were also plotted to examine if different starting lengths (different lengths of strip) had any effect on the turns per cm. at which wrapping occurs. Once again observations were made with different widths of rubber strips and it will be seen from Fig. 4.3 that except for the 5 cm. long strip all the other lengths wrap at the same number of turns per cm. The curve for 5 cm. long strip seem to deviate slightly from the other curves and may be attributed to the fact that as it is very difficult to obtain a perfectly uniform width of ribbon all throughout the length of the strip, any narrowpart in the width will show a magnified effect in smaller lengths. It can also be seen from the same figure that as the ribbon width decreases the turns per cm. at which wrapping occurs also decreases.

EXPERIMENTAL CURVE FOR TWISTING OF RUBBER STRIP AT 5gms. TENSION.

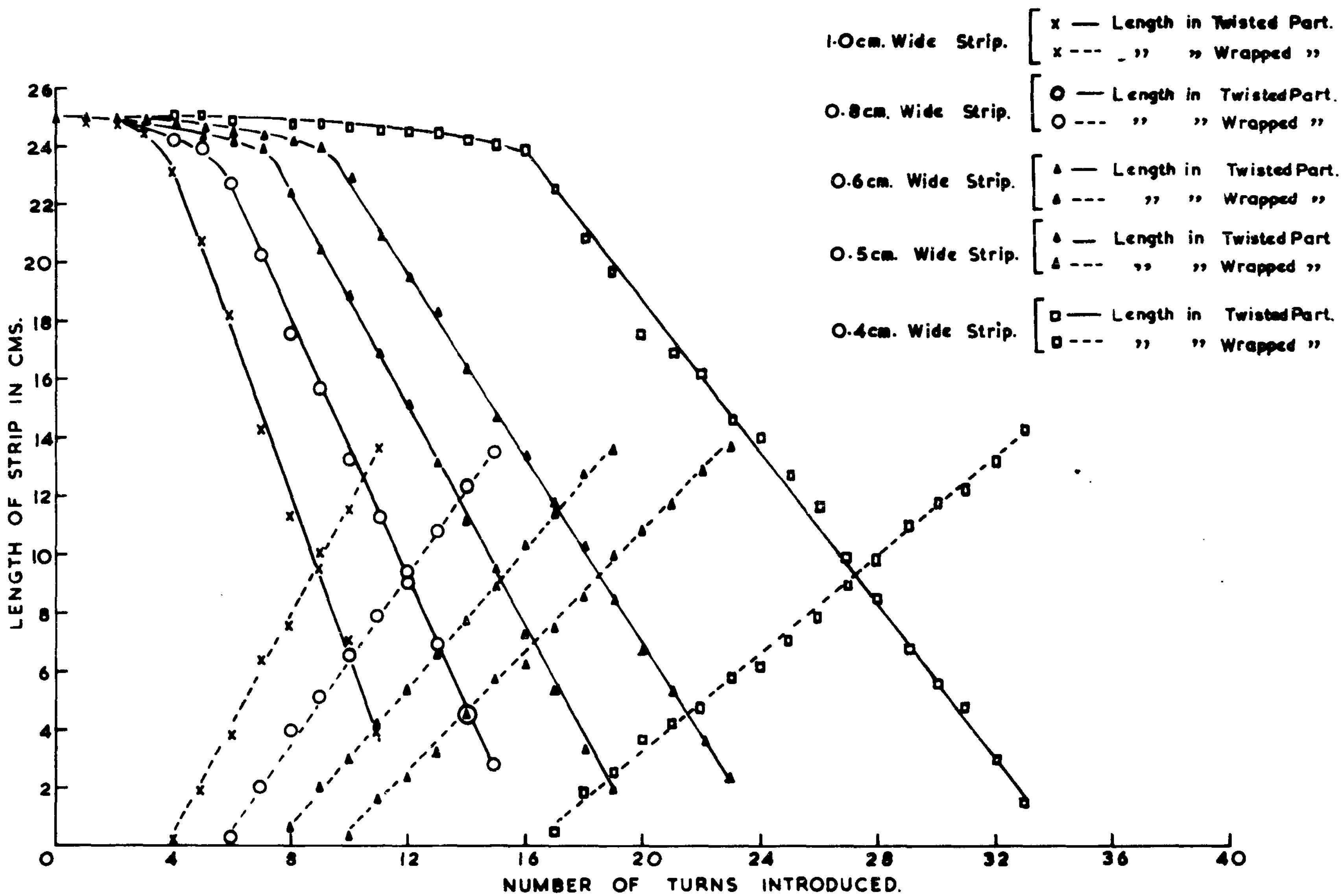
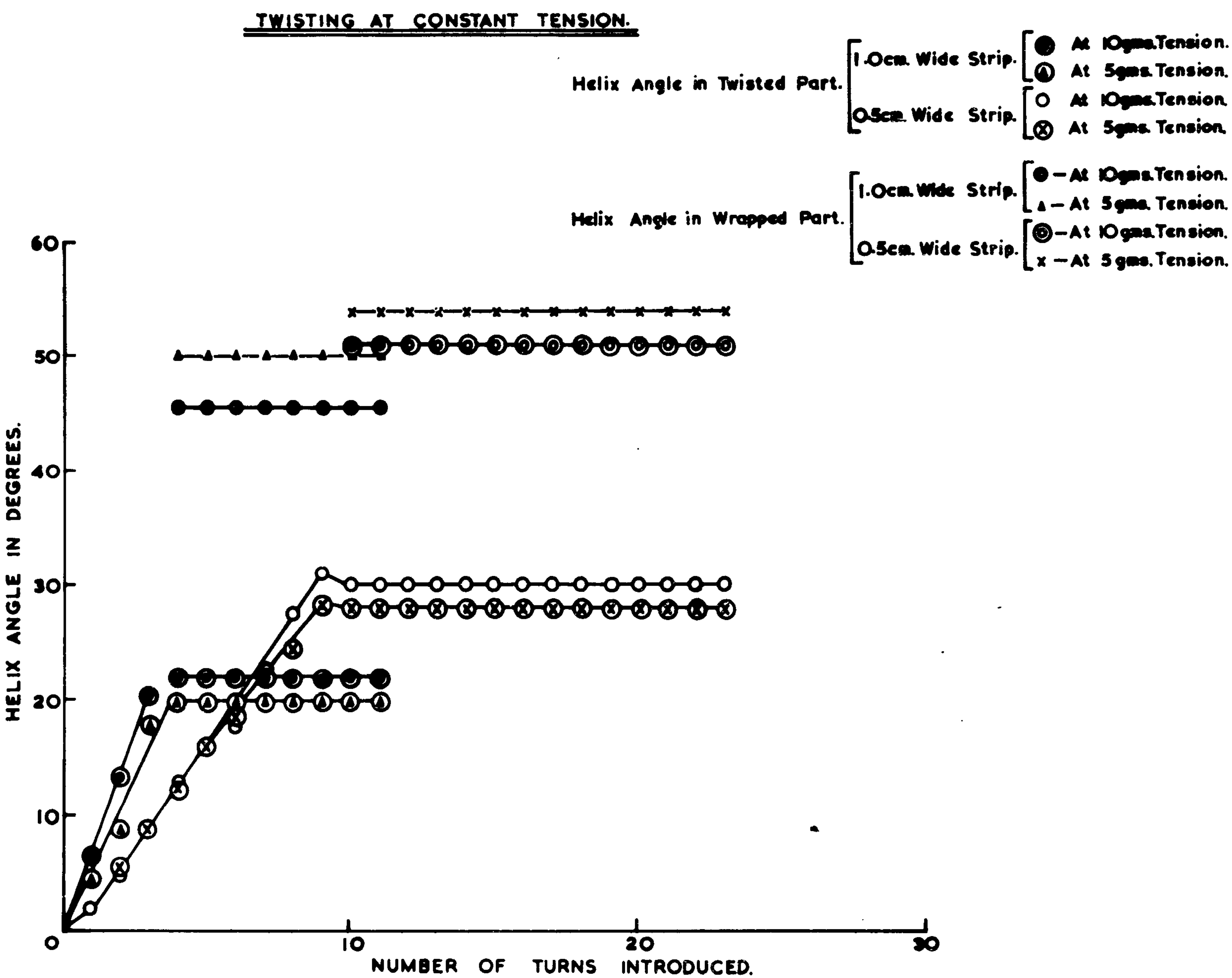


FIG. 4.5.



TWISTING AT CONSTANT TENSION.
Helix Angle in Twisted Part.
1.0cm. Wide Strip.
At 10gms.Tension.
At 5gms. Tension.
0.5cm. Wide Strip.
At 10gms.Tension.
At 5gms. Tension.
Helix Angle in Wrapped Part.
1.0cm. Wide Strip.
– At 10gms.Tension.
– At 5gms. Tension.
0.5cm. Wide Strip.
– At 10gms.Tension.
x – At 5gms. Tension.
HELIX ANGLE IN DEGREES.
0
10
20
30
40
50
60
10
20
30
NUMBER OF TURNS INTRODUCED.

FIG. 4. 6.

TWISTING AT CONSTANT TENSION.

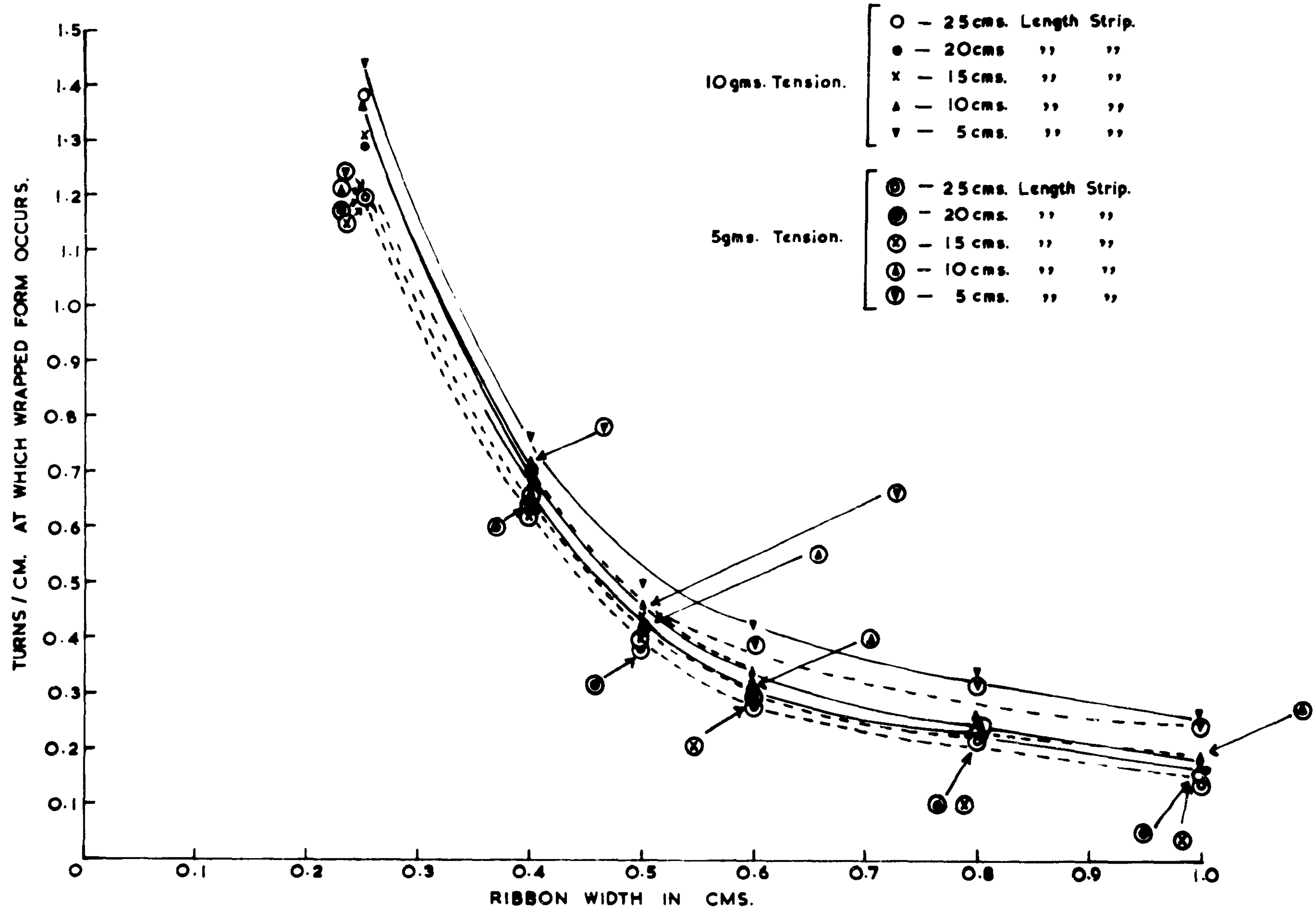


FIG. 4.7.

In Fig. 4.4 helix angle in the wrapped part and helix angle in the twisted part, (after wrapping has started), have been plotted. It can be seen that the helix angle decreases with the increase in the ribbon width, showing that in narrower ribbons the wrapped form is obtained much later than in wider ribbons, i.e., a greater number of turns have to be introduced in a narrower ribbon to obtain a wrapped form than in the wider ribbon.

4.3 Twisting at constant tension

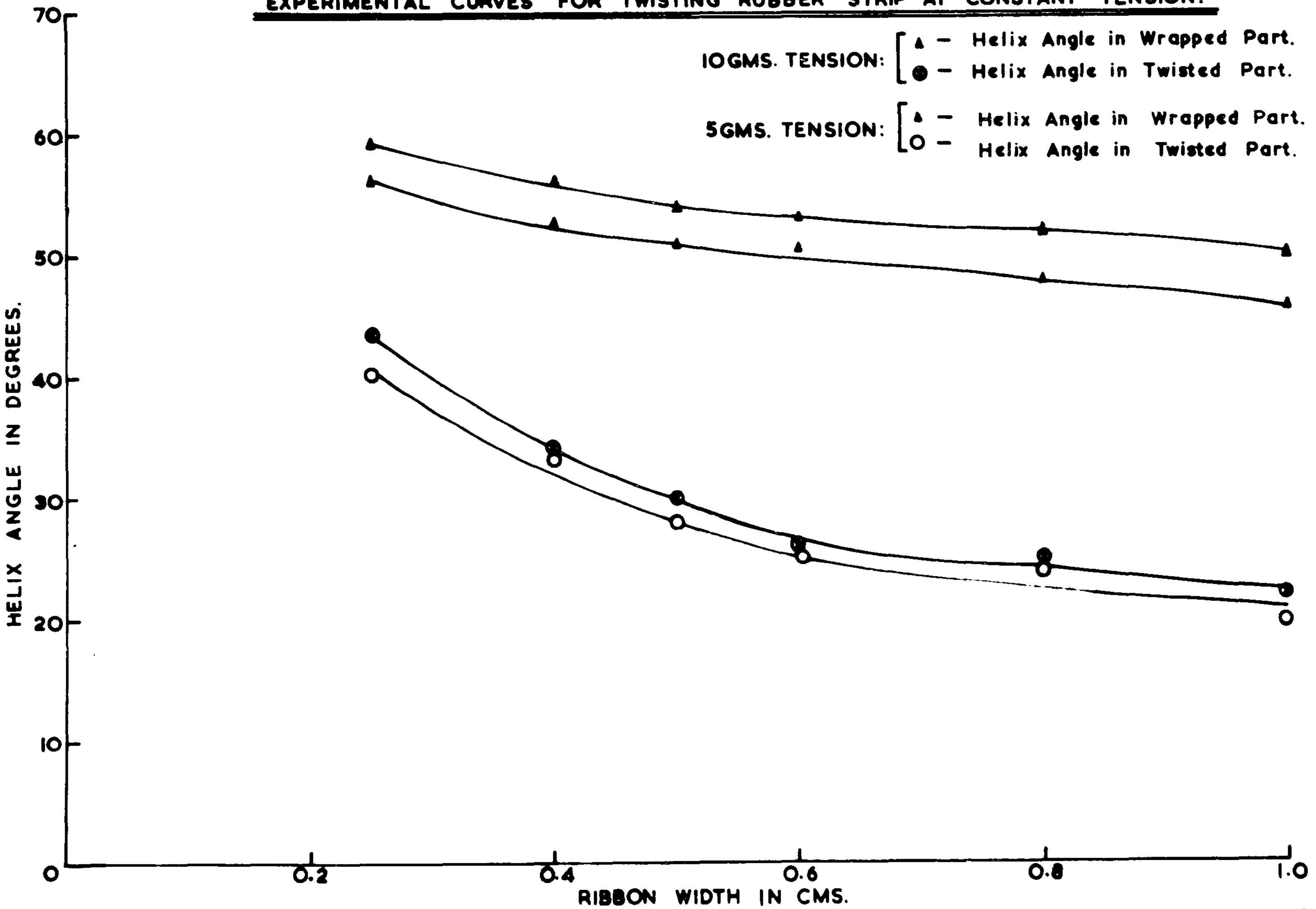
Observations for helix angle and length of strip in the twisted and wrapped part were also made under constant tension. The general appearance of curves in Fig. 4.5 to Fig. 4.8 appear to be the same as that of the curves in Fig. 4.1 to Fig. 4.4 (twisted at constant length) but on closely observing them the differences can be marked out.

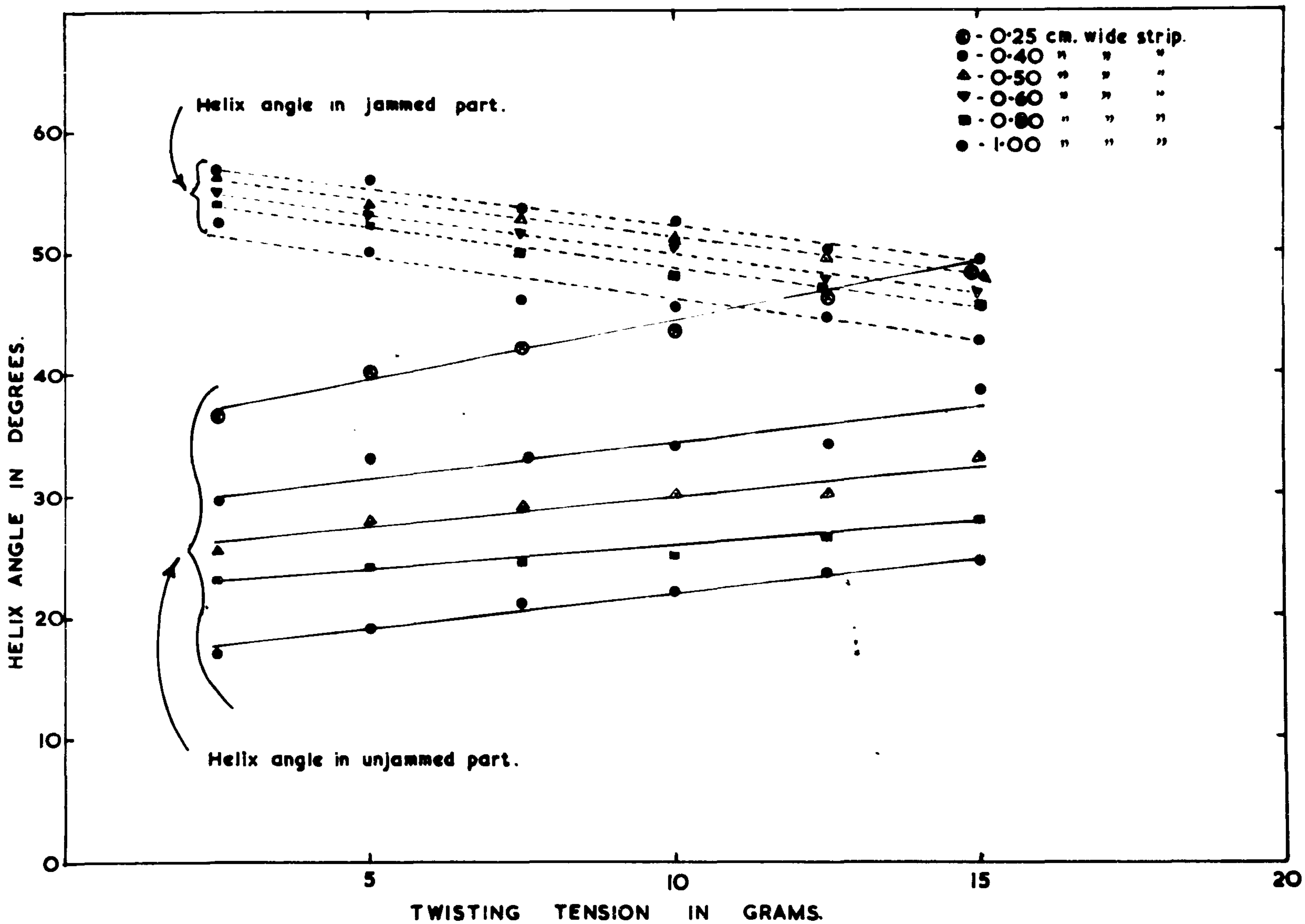
The initial portion of the curve for the twisted part in Fig. 4.5

is not parallel to the abscissae as is the case in Fig. 4.1. This is due to the contraction in length of the strip during twisting. This contraction in the length of the strip is more when the strips are twisted at a lower tension and is shown in Fig. 4.11, where the percentage contraction in length has been plotted against the number of turns. It can be seen that the percentage contraction in length in the twisted part is very small, but once the wrapped part has started forming the percentage contraction in length increases rapidly. Although experimental values for only two ribbon widths (1 cm.wide and 0.4 cm. wide strip) has been plotted in Fig. 4.11, the percentage contraction values for other widths have been tabulated in the tables (Table 4.8 to Table 4.18). It can also be observed from the Fig. 4.11 that the percentage contraction in length for the same number of turns in the wider strip is much higher than the percentage contraction in the narrower strips.

FIG. 4. 8.

EXPERIMENTAL CURVES FOR TWISTING RUBBER STRIP AT CONSTANT TENSION.





O·25 cm. wide strip.
O·40 " " "
O·50 " " "
O·60 " " "
O·80 " " "
1·OO " " "
Helix angle in jammed part.
Helix angle in unjammed part.
HELIX ANGLE IN DEGREES.
TWISTING TENSION IN GRAMS.

FIG. 4.9.

FIG. 4. 10.

EXPERIMENTAL CURVES FOR TWISTING RUBBER STRIPS.

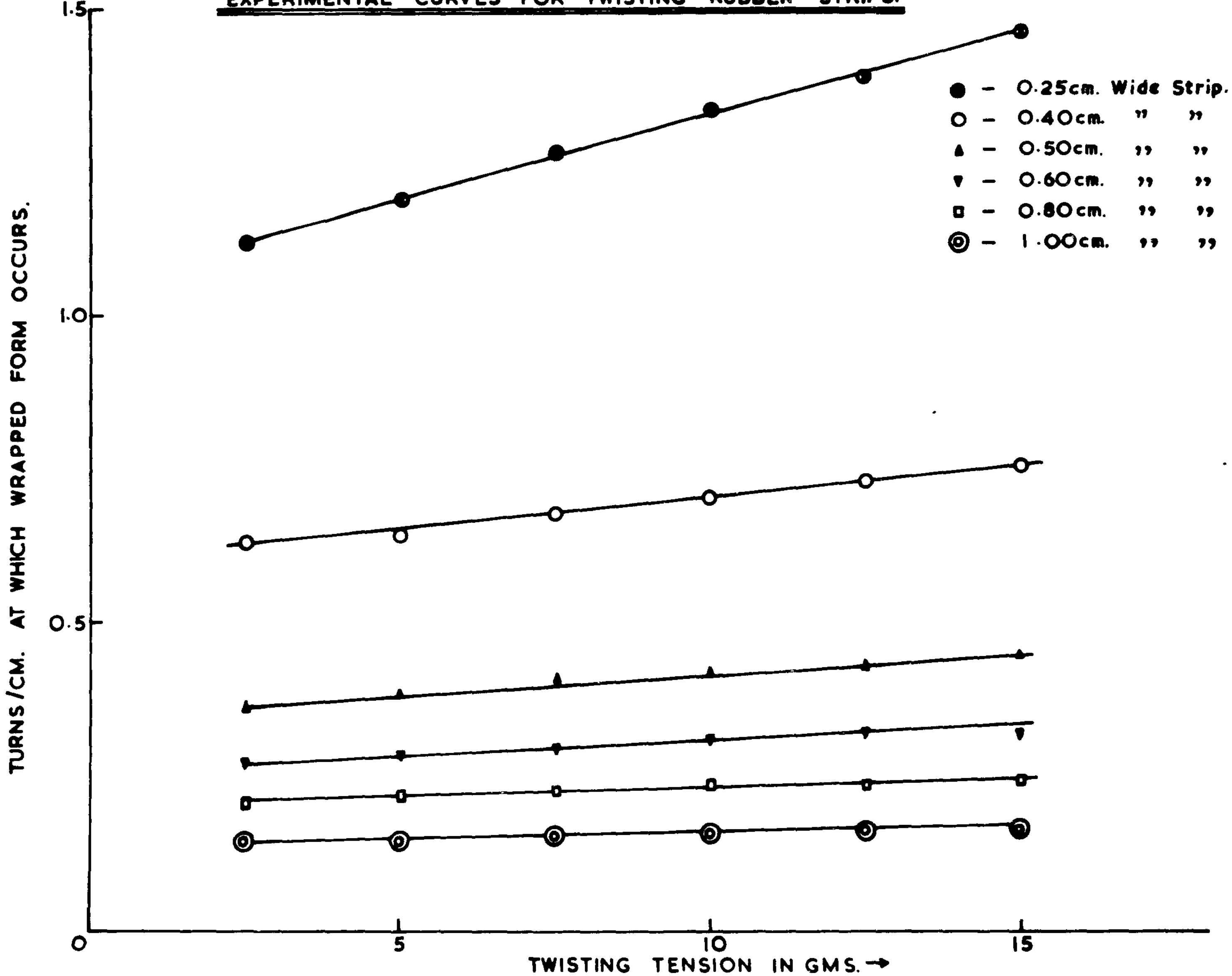
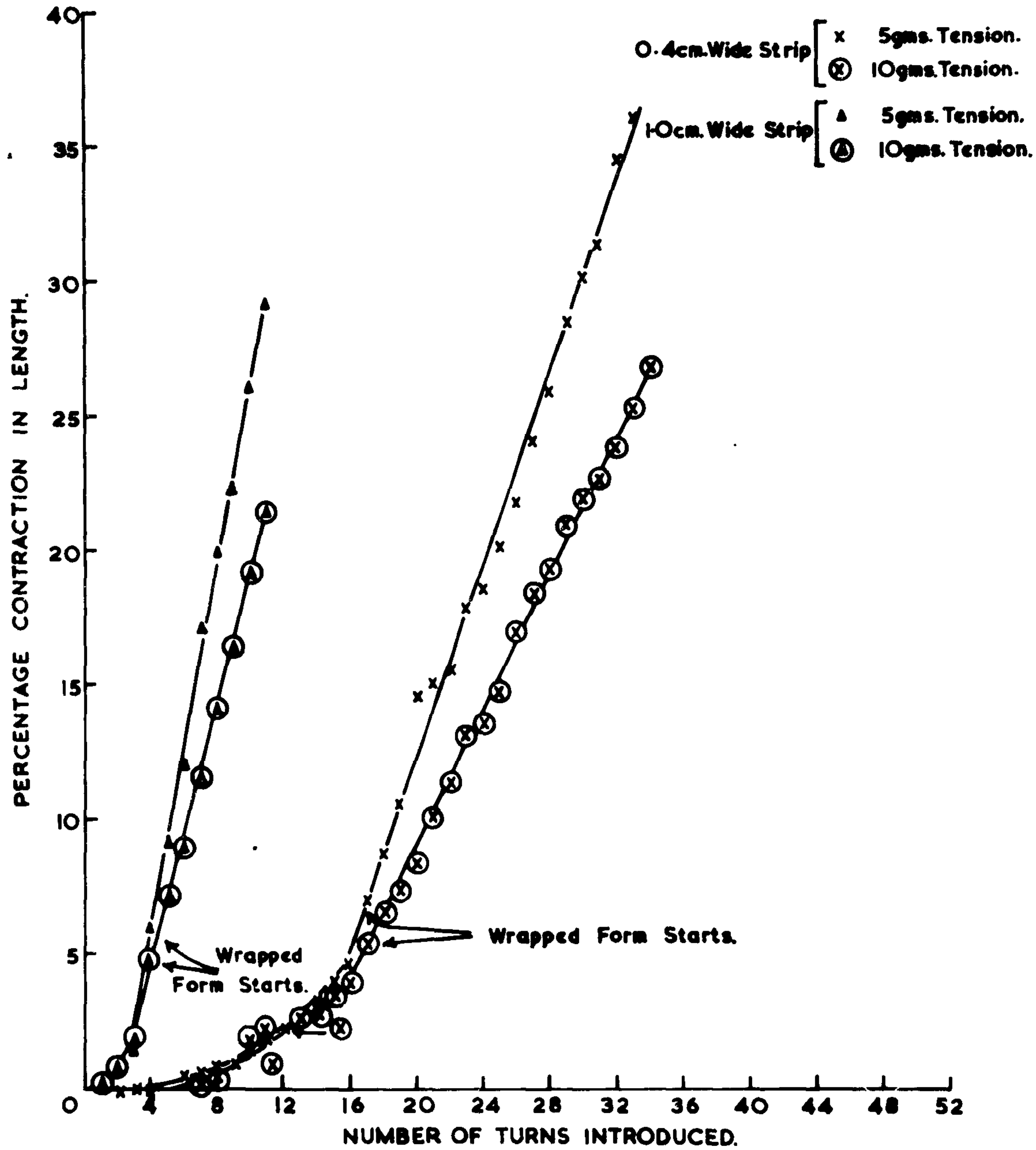


FIG. 4. 11.

TWISTING AT CONSTANT TENSION.



Comparing the Figures 4.1 and 4.5 we find that at a certain value of twist for a particular ribbon width, the length of strip in the wrapped part, twisted at constant length, is more than the length in the wrapped part when twisted under constant tension. This is because of the reason that when twisted at constant length the tension on the ribbon increases rapidly with twist and the length cannot contract.

It will also be observed that the helix angle in the twisted and wrapped part in Fig. 4.6 is higher than the corresponding values in Fig. 4.2.

By looking at the Fig. 4.8 and Fig. 4.9 it can be seen that the helix angle in the twisted part increases as the tension increases whereas the helix angle in the wrapped part decreases as the tension increases. This is true for all the ribbon widths and is shown in Fig. 4.9.

Fig. 4.7 and Fig. 4.10 were plotted to observe the effect of twisting tension on the turns per cm. at which wrapping occurs. It can be observed from those figures that the turns/cm. at which wrapping occurs increases as the tension increases. The rate of increase of wrapping turns per cm. with tension is more or less the same for all the ribbon widths except for the ribbon of 0.25 cm. width as is indicated by the slope of the curves. In the case of the 0.25 cm. wide ribbon it seems that the rate of increase is slightly higher.

4.4 Measurement of properties of strip

In order to provide the necessary figures to substitute in the theoretical relation described in the next Chapter, elastic moduli and other parameter of the rubber were determined using the methods described in the previous Chapter.

TABLE 4. 1.

Twisting at Constant Length - 25 cms.

Ribbon Width - 1 cm.

Number of Turns Introduced	Turns/cm	Turns/cm in the Twisted Part	Length of Strip Wrapped along the Axis of Yarn Cms.	Number of Turns in The Wrapped Part	Turns/cm in the Wrapped Part along the Axis.	Helix Angle in the Twisted Part	Helix Angle in the Wrapped Part
0	0	0	0	0	0	0	0
1	0.04	0.04	-	-	-	5.5°	-
2	0.08	0.08	-	-	-	12.5°	-
3	0.12	0.12	-	-	-	19.5°	-
4	0.16	0.13	2.7	1	0.37	"	30°
5	0.20	0.14	4.5	2	0.44	"	"
6	0.24	0.12	9.0	4	0.44	"	"
7	0.28	0.16	13.0	5	0.38	"	"
8	0.32	0.12	17.0	7	0.41	"	"
9	0.36	0.18	19.5	8	0.41	"	"
10	0.40	0.40	22.5	9	0.40	"	"

TABLE 4. 2.

Twisting at Constant Length - 25 cms.

Ribbon Width - 0.8 cms.

Number of Turns Introduced	Turns / cm	Turns/cm in the Twisted Part	Length of Strip Wrapped Along the Axis of Yarn Cms.	Number of Turns In The Wrapped Part	Turns/cm in the Wrapped Part Along the Axis	Helix Angle in the Twisted Part	Helix Angle in the Wrapped Part
0	0	0	0	0	0	0	0
1	0.04	0.04	-	-	-	4.5°	-
2	0.08	0.08	-	-	-	9.5°	-
3	0.12	0.12	-	-	-	15.0°	-
4	0.16	0.16	-	-	-	18.5°	-
5	0.20	0.20	-	-	-	22.5°	-
6	0.24	0.24	-	-	-	24.0°	-
7	0.28	0.241	2.2	1.5	0.682	22.0°	33°
8	0.32	0.245	4.6	3.0	0.652	"	"
9	0.36	0.247	6.8	4.5	0.662	"	"
10	0.40	0.256	9.4	6.0	0.638	"	"
11	0.44	0.263	11.7	7.5	0.641	"	"
12	0.48	0.258	13.4	9.0	0.672	"	"
13	0.52	0.284	16.2	10.5	0.648	"	"
14	0.56	0.286	18.0	12.0	0.667	"	"
15	0.60	0.326	20.4	13.5	0.662	"	"
16	0.64	0.500	22.0	14.5	0.659	"	"

TABLE 4. 3.

Twisting at Constant Length - 25 cms.

Ribbon Width - 0.5 cms.

Number of Turns Introduced	Turns/cm	Turns/cm in Twisted Part	Length of Strip Wrapped Along the Axis of Yarn Cms.	Number of Turns In The Wrapped Part	Turns/CM in the Wrapped Part Along the Axis	Helix Angle in the Twisted Part	Helix Angle in the Wrapped Part
0	0	0	0	0	0		0
1	0.04	0.04	-	-	-	4°	-
2	0.08	0.08	-	-	-	7°	-
3	0.12	0.12	-	-	-	10.5°	-
4	0.16	0.16	-	-	-	14.5°	-
5	0.20	0.20	-	-	-	17.0°	-
6	0.24	0.24	-	-	-	20.5°	-
7	0.28	0.28	-	-	-	23.0°	-
8	0.32	0.32	-	-	-	26.5°	-
9	0.36	0.36	-	-	-	28.5°	-
10	0.40	0.40	-	-	-	31.0°	-
11	0.44	0.44	-	-	-	32.0°	-
12	0.48	0.422	1.3	2	1.538	28.0°	43.0°
13	0.52	0.444	2.5	3	1.200	"	"
14	0.56	0.469	3.7	4	1.081	"	"
15	0.60	0.450	5.0	6	1.200	"	"
16	0.64	0.430	6.4	8	1.250	"	"
17	0.68	0.424	8.5	10	1.176	"	"
18	0.72	0.446	9.3	11	1.182	"	"
19	0.76	0.483	10.5	12	1.143	"	"
20	0.80	0.518	11.5	13	1.130	"	"
21	0.84	0.500	13.0	15	1.154	"	"
22	0.88	0.476	14.5	17	1.172	"	"
23	0.92	0.555	16.0	18	1.125	"	"
24	0.96	0.400	17.5	21	1.200	"	"
25	1.00	0.469	18.6	22	1.183	"	"
26	1.04	0.600	20.0	23	1.150	"	"
27	1.08	0.667	22.0	25	1.136	"	"

TABLE 4. 4.

Twisting at Constant Length - 25 cms.

Ribbon Width - 0.4 cms.

Number of Turns Introduced	Turns/CM	Turns/CM in the Twisted Part	Length of Strip Wrapped Along the Axis of Yarn Cms.	Number of Turns in The Wrapped Part	Turns/CM in the Wrapped Part Along the Axis	Helix Angle in the Twisted Part	Helix Angle in the Wrapped Part
0	0	0	0	0	0	0	0
1	0.04	0.04	-	-	-	1^{o}	-
2	0.08	0.08	-	-	-	3.5^{o}	-
3	0.12	0.12	-	-	-	5.5^{o}	-
4	0.16	0.16	-	-	-	8.5^{o}	-
5	0.20	0.20	-	-	-	12.0^{o}	-
6	0.24	0.24	-	-	-	14.0^{o}	-
7	0.28	0.28	-	-	-	14.5^{o}	-
8	0.32	0.32	-	-	-	18.0^{o}	-
9	0.36	0.36	-	-	-	19.5^{o}	-
10	0.40	0.40	-	-	-	22.5^{o}	-
11	0.44	0.44	-	-	-	23.5^{o}	-
12	0.48	0.48	-	-	-	26.0^{o}	-
13	0.52	0.52	-	-	-	28.0^{o}	-
14	0.56	0.56	-	-	-	30.0^{o}	-
15	0.60	0.60	-	-	-	32.0^{o}	-
16	0.64	0.64	-	-	-	34.0^{o}	-
17	0.68	0.68	-	-	-	35.0^{o}	-
18	0.72	0.72	-	-	-	35.5^{o}	-
19	0.76	0.76	-	-	-	36.0^{o}	-
20	0.80	0.729	1.7	3	1.765	32.0^{o}	49.0^{o}
21	0.84	0.740	2.7	4.5	1.667	"	"
22	0.88	0.755	3.8	6.0	1.579	"	"
23	0.92	0.752	4.4	7.5	1.704	"	"
24	0.96	0.767	5.2	9.0	1.731	"	"
25	1.00	0.703	5.8	11.5	1.883	"	"

Table 4.4. (Continued)

Number of Turns Introduced	Turns/CM	Turns/CM in the Twisted Part	Length of Strip Wrapped Along the Axis of Yarn Cms.	Number of Turns in The Wrapped Part	Turns/CM in the Wrapped Part Along the Axis	Helix Angle in the Twisted Part	Helix Angle in the Wrapped Part
26	1.04	0.722	7.0	13.0	1.857	32.0^{o}	49.0^{o}
27	1.08	0.748	8.3	14.5	1.747	"	"
28	1.12	0.728	9.2	16.5	1.793	"	"
29	1.16	0.757	9.8	17.5	1.786	"	"
30	1.20	0.753	10.4	19.0	1.827	"	"
31	1.24	0.786	11.0	20.0	1.818	"	"
32	1.28	0.781	12.2	22.0	1.803	"	"
33	1.32	0.775	13.4	24.0	1.791	"	"
34	1.36	0.833	14.2	25.0	1.761	"	"
35	1.40	0.800	15.0	27.0	1.800	"	"
36	1.44	0.786	16.1	29.0	1.801	"	"
37	1.48	0.764	17.8	31.5	1.769	"	"
38	1.52	0.821	18.3	32.5	1.776	"	"
39	1.56	0.776	19.2	34.5	1.797	"	"
40	1.60	0.869	20.4	36.0	1.765	"	"
41	1.64	1.029	21.6	37.5	1.736	"	"
42	1.68	0.893	22.2	39.5	1.779	"	"
43	1.72	1.000	23.0	41.0	1.783	"	"

TABLE 4.5.

TWISTING AT CONSTANT LENGTH - 25 CMS.

RIBBON WIDTH - 0.25 CMS.

Number of Turns Introduced	Turns / cm	Turns/cm In The Twisted Part	Length of Strip Wrapped Along the Axis of Yarn Cms.	Number of Turns in The Wrapped Part	Turns/cm in the Wrapped Part Along the Axis	Helix Angle in the Twisted Part	Helix Angle in The Wrapped Part
0	0	0	0	0	0	0	0
1	0.04	0.04	-	-	-	1.5^{o}	-
2	0.08	0.08	-	-	-	3.0^{o}	-
3	0.12	0.12	-	-	-	5.0^{o}	-
4	0.16	0.16	-	-	-	6.5^{o}	-
5	0.20	0.20	-	-	-	7.5^{o}	-
6	0.24	0.24	-	-	-	9.5^{o}	-
7	0.28	0.28	-	-	-	11.5^{o}	-
8	0.32	0.32	-	-	-	13.0^{o}	-
9	0.36	0.36	-	-	-	13.5^{o}	-
10	0.40	0.40	-	-	-	15.0^{o}	-
11	0.44	0.44	-	-	-	16.5^{o}	-
12	0.48	0.48	-	-	-	18.5^{o}	-
13	0.52	0.52	-	-	-	20.0^{o}	-
14	0.56	0.56	-	-	-	22.0^{o}	-
15	0.60	0.60	-	-	-	23.0^{o}	-
16	0.64	0.64	-	-	-	24.5^{o}	-
17	0.68	0.68	-	-	-	26.0^{o}	-
18	0.72	0.72	-	-	-	27.0^{o}	-
19	0.76	0.76	-	-	-	29.5^{o}	-
20	0.80	0.80	-	-	-	31.5^{o}	-
21	0.84	0.84	-	-	-	33.5^{o}	-
22	0.88	0.88	-	-	-	35.5^{o}	-
23	0.92	0.92	-	-	-	36.5^{o}	-
24	0.96	0.96	-	-	-	38.5^{o}	-

TABLE 4.5 (Continued)

Number of Turns Introduced	Turns / CM	Turns/CM in the Twisted Part	Length of Strip Wrapped Along the Axis of Yarn Cms.	Number of Turns In The Wrapped Part	Turns/CM in the Wrapped Part Along the Axis	Helix Angle in the Twisted Part	Helix Angle in the Wrapped Part
25	1.00	1.00	-	-	-	39.5°	-
26	1.04	1.04	-	-	-	40.0°	-
27	1.08	1.08	-	-	-	41.0°	-
28	1.12	1.12	-	-	-	41.5°	-
29	1.16	1.16	-	-	-	42.0°	-
30	1.20	1.20	-	-	-	42.5°	-
31	1.24	1.16	0.80	3	3.53	37.0°	54.0°
32	1.28	1.15	1.60	5	3.12	"	"
33	1.32	1.20	1.60	6	3.03	"	"
34	1.36	1.20	2.50	7	2.80	"	"
35	1.40	1.22	2.80	8	2.86	"	"
36	1.44	1.22	3.70	10	2.70	"	"
37	1.48	1.23	3.90	11	2.82	"	"
38	1.52	1.25	4.20	12	2.86	"	"
39	1.56	1.26	5.10	14	2.75	"	"
40	1.60	1.27	5.30	15	2.83	"	"
41	1.64	1.26	5.90	17	2.86	"	"
42	1.68	1.25	6.60	19	2.88	"	"
43	1.72	1.26	7.60	21	2.76	"	"
44	1.76	1.24	8.10	23	2.84	"	"
45	1.80	1.27	8.50	24	2.82	"	"
46	1.84	1.35	8.70	25	2.76	"	"
47	1.88	1.35	8.70	26	2.86	"	"
48	1.92	1.36	9.60	27	2.81	"	"
49	1.96	1.29	11.10	31	2.79	"	"
50	2.00	1.24	12.90	35	2.71	"	"
51	2.04	1.26	13.10	36	2.75	"	"

TABLE 4.5 (Continued)

Number of Turns Introduced	Turns / CM	Turns / CM in the Twisted Part	Length of Strip Wrapped Along the Axis of Yarn cms	Number of Turns in the Wrapped Part	Turns/CM in the Wrapped Part Along the Axis	Helix Angle in the Twisted Part	Helix Angle In the Wrapped Part
52	2.08	1.32	13.60	37	2.72	37.0°	54.0°
53	2.12	1.27	14.00	39	2.79	"	"
54	2.16	1.25	14.60	41	2.81	"	"
55	2.20	1.26	15.50	43	2.77	"	"
56	2.24	1.28	16.40	45	2.74	"	"
57	2.28	1.23	16.90	47	2.78	"	"
58	2.32	1.26	17.10	48	2.81	"	"
59	2.36	1.31	17.40	49	2.82	"	"
60	2.40	1.29	18.80	52	2.77	"	"
61	2.44	1.38	19.20	53	2.76	"	"
62	2.48	1.25	19.40	55	2.84	"	"
63	2.52	1.32	19.70	56	2.84	"	"
64	2.56	1.40	20.00	57	2.85	"	"
65	2.60	1.52	20.40	58	2.84	"	"
66	2.64	1.39	20.70	60	2.90	"	"
67	2.68	1.46	20.90	61	2.92	"	"
68	2.72	1.54	21.10	62	2.94	"	"
69	2.76	2.00	23.50	66	2.81	"	"
70	2.80	2.14	23.60	67	2.84	"	"
71	2.84	2.31	23.70	68	2.87	"	"
72	2.88		25.00	72	2.88		

TABLE 4.6

TWISTING AT CONSTANT LENGTH

Length of Strip	Wrapping Occurs at -													
	For 0.25 cm Width		For 0.40 cm Width		For 0.50 cm Width		For 0.60 cm Width		For 0.75 cm Width		For 0.80 cm Width		For 1.0 cm Width	
	Number or Turns	Turns/ CM	Number of Turns	Turns/ CM	Number of Turns	Turns/ CM	Number of Turns	Turns/ CM	Number of Turns	Turns/ CM	Number of Turns	Turns/ CM	Number of Turns	Turns/ CM
25 cms	29.4	1.176	19.3	0.772	11.9	0.476	9.3	0.372	6.8	0.272	6.5	0.260	4.2	0.168
20 cms	24.0	1.200	15.4	0.770	9.6	0.480	7.3	0.365	5.4	0.270	5.3	0.265	3.4	0.170
15 cms	17.9	1.190	12.0	0.800	7.4	0.493	6.0	0.400	4.2	0.280	4.1	0.273	2.5	0.166
10 cms	11.9	1.190	8.0	0.800	4.9	0.490	4.3	0.430	3.0	0.300	2.9	0.290	1.9	0.190
5 cms	6.0	1.200	4.2	0.840	2.6	0.520	2.3	0.470	1.8	0.360	1.7	0.340	1.0	0.200

TABLE 4.7

Ribbon Width Cms	Helix Angle in the	
	Twisted Part	Wrapped Part
0.25	37°	54°
0.40	32°	49°
0.50	28°	43°
0.60	26.5°	38.5°
0.75	22.5°	35.0°
0.80	22.0°	33.0°
1.00	19.5°	30.0°

TABLE 4.8

TWISTING AT CONSTANT TENSION - 1 cm WIDE STRIP - 25 cm LENGTH

TWISTING TENSION - 10 gms. POSITION OF TWISTING JAW - 3.0 cms.

Number of Turns Introduced	Turns/CM in the Twisted Part	Length of Strip Wrapped Along the Axis of Yarn cms	Number of Turns In the Wrapped Part	Turns/CM in the Wrapped Part	Helix Angle in the Twisted Part	Helix Angle in the Wrapped Part Along the Axis	Reading on the Scale As Length Contracts cms	Contraction cms	Contraction %	Length of Strip in Twisted Part cms
0	0	0	0	0	0	0	28.0	0	0	25
1	0.040	-	-	-	6.5°	-	27.95	0.05	0.20	24.95
2	0.081	-	-	-	13.5°	-	27.80	0.20	0.80	24.80
3	0.122	-	-	-	20.5°	-	27.55	0.45	1.80	24.55
4	0.149	0.3	0.5	1.667	22.0	45.5	26.80	1.20	4.80	23.50
5	0.166	2.1	1.5	0.714	"	"	26.20	1.80	7.20	21.10
6	0.185	3.8	2.5	0.658	"	"	25.75	2.25	90.00	18.95
7	0.194	6.6	4.0	0.606	"	"	25.10	2.90	11.60	15.50
8	0.198	8.5	5.5	0.647	"	"	24.45	3.55	14.20	12.65
9	0.250	10.2	6.5	0.637	"	"	23.90	4.10	16.40	10.00
10	0.259	12.5	8.0	0.640	"	"	23.20	4.80	19.20	7.70
11	0.312	14.8	9.5	0.642	"	"	22.60	5.40	21.60	4.80

TABLE 4.9

TWISTING AT CONSTANT TENSION - 1 CM WIDE STRIP - 25 CM LENGTH

TWISTING TENSION - 5 GMS POSITION OF TWISTING JAW - 3.0 CMS

Number of Turns Introduced	Turns/CM In the Twisted Part	Length of Strip Wrapped Along the Axis of Yarn. cms	Number of Turns In the Wrapped Part	Turns/CM in the Wrapped Part	Helix Angle in the Twisted Part	Helix Angle in the Wrapped Part Along the Axis	Reading on the Scale as Length contracts cms	Contraction cms	% Contraction	Length of Strip In Twisted Part cms
0		0	0	0	0	0	28.0	0	0	25
1	0.040	-	-	-	4.5°	-	27.95	0.05	0.20	24.95
2	0.081	-	-	-	9.0°	-	27.80	0.20	0.80	24.80
3	0.122	-	-	-	18.0°	-	27.60	0.40	1.60	24.60
4	0.142	0.3	0.5	1.667	20.0°	50.0°	26.50	1.50	6.00	23.20
5	0.168	1.9	1.5	0.789	"	"	25.70	2.30	9.20	20.80
6	0.192	3.8	2.5	0.658	"	"	25.00	3.00	12.00	18.20
7	0.210	6.4	4.0	0.625	"	"	23.70	4.30	17.20	14.30
8	0.219	8.6	5.5	0.724	"	"	23.00	5.00	20.00	11.40
9	0.200	9.4	7.0	0.745	"	"	22.40	5.60	22.40	10.00
10	0.286	11.5	8.0	0.696	"	"	21.50	6.50	26.00	7.00
11	0.375	13.7	9.5	0.693	"	"	20.70	7.30	29.20	4.00

TABLE 4.10

TWISTING AT CONSTANT TENSION - 0.8cm WIDE STRIP - 25 CM LENGTH

TWISTING TENSION - 10 GMS   POSITION OF TWISTING JAW - 3 CMS

Number of Turns Introduced	Turns/CM in the Twisted Part	Length of Strip Wrapped along the Axis of Yarn cms	Number of Turns In the Wrapped Part	Turns/cm in the Wrapped Part	Helix Angle in the Twisted Part	Helix Angle in the Wrapped Part along the Axis	Reading on the Scale as Length Contracts cms	Contraction cms	% Contraction	Length of Strip in Twisted Part cms
0	0	0	0	0	0	0	28	0	0	25
1	0.040	-	-	-	4.5°	-	28	0	0	25
2	0.080	-	-	-	9.0°	-	27.95	0.05	0.20	24.95
3	0.121	-	-	-	14.5°	-	27.75	0.25	1.00	24.75
4	0.163	-	-	-	19.5°	-	27.55	0.45	1.80	24.55
5	0.205	-	-	-	24.5°	-	27.35	0.65	2.60	24.35
6	0.235	0.3	0.5	1.667	25°	48°	26.70	1.30	5.20	23.40
7	0.237	2.0	2.0	1.000	"	"	26.10	1.90	7.60	21.10
8	0.238	3.7	3.5	0.946	"	"	25.60	2.40	9.60	18.90
9	0.237	5.2	5.0	0.962	"	"	25.05	2.95	11.80	16.85
10	0.286	6.4	6.0	0.937	"	"	24.40	3.60	14.40	15.00
11	0.239	8.4	8.0	0.952	"	"	23.95	4.05	16.20	12.55
12	0.278	9.7	9.0	0.928	"	"	23.50	4.50	18.00	10.80
13	0.301	11.6	10.5	0.905	"	"	22.90	5.10	20.40	8.30
14	0.303	12.8	12.0	0.938	"	"	22.40	5.60	22.40	6.60
15	0.349	14.5	13.5	0.931	"	"	21.80	6.20	24.80	4.30
16	0.385	15.8	15.0	0.949	"	"	21.40	6.60	26.40	2.60

TABLE 4.11

TWISTING AT CONSTANT TENSION - 0.8 cm WIDE STRIP - 25 cm LENGTH

TWISTING TENSION - 5 GMS. POSITION OF TWISTING JAW - 3.0 cms

Number of Turns Introduced	Turns/cm in the Twisted Part	Length of Strip Wrapped Along the Axis of Yarn cms	Number of Turns in the Wrapped Part	Turns/cm in the Wrapped Part	Helix angle in the twisted part	Helix Angle in the Wrapped Part Along the Axis	Reading on the Scale as length Contracts cms	Contraction cms	% Contraction	Length of Strip in Twisted Part cms
0	0	0	0	0	0	0	27.90	0	0	24.90
1	0.040	-	-	-	5°	-	27.85	0.05	0.20	24.85
2	0.081	-	-	-	10.5°	-	27.75	0.15	0.60	24.75
3	0.122	-	-	-	15.5°	-	27.55	0.35	1.40	24.55
4	0.165	-	-	-	20.5°	-	27.30	0.60	2.40	24.30
5	0.208	-	-	-	25.5°	-	27.05	0.85	3.40	24.05
6	0.241	0.3	0.5	1.667	24.0°	52.5°	26.10	1.80	7.20	22.80
7	0.246	2.0	2.0	1.000	"	"	25.30	2.60	10.40	20.30
8	0.227	4.0	4.0	1.000	"	"	24.60	3.30	13.20	17.60
9	0.255	5.1	5.0	0.980	"	"	23.80	4.10	16.40	15.70
10	0.263	6.5	6.5	1.000	"	"	22.80	5.10	20.40	13.30
11	0.265	7.9	8.0	1.012	"	"	22.20	5.70	22.80	11.30
12	0.275	9.4	9.5	1.010	"	"	21.50	6.40	25.60	9.10
13	0.290	10.8	11.0	1.018	"	"	20.70	7.20	28.80	6.90
14	0.326	12.3	12.5	1.016	"	"	19.90	8.00	32.00	4.60
15	0.536	13.5	13.5	1.000	"	"	19.30	8.70	34.80	2.80

TABLE 4.12

TWISTING AT CONSTANT TENSION - 0.6 cm WIDE STRIP - 25 cm LENGTH

TWISTING TENSION - 10 GMS POSITION OF TWISTING JAW - 3.0 cms

Number of Turns Introduced	Turns/cm in the Twisted Part	Length of Strip Wrapped along the axis of Yarn cms	Number of Turns in the Wrapped Part	Turns/cm in the Wrapped Part	Helix angle in the Twisted Part	Helix angle in the Wrapped Part Along the Axis	Reading on the Scale as length contracts cms	Contraction cms	% Contraction	Length of Strip in Twisted Part cms
0	0	0	0	0	0	0	28.1	0	0	25.1
1	0.040	-	-	-	2.0°	-	28.1	0	0	25.1
2	0.080	-	-	-	6.5°	-	28.0	0.10	0.40	25.0
3	0.120	-	-	-	10.5°	-	27.9	0.20	0.80	24.9
4	0.162	-	-	-	15.0°	-	27.75	0.35	1.40	24.75
5	0.203	-	-	-	19.5°	-	27.60	0.50	2.00	24.60
6	0.246	-	-	-	23.5°	-	27.40	0.70	2.80	24.40
7	0.289	-	-	-	26.5°	-	27.20	0.90	3.60	24.20
8	0.306	0.7	1.0	1.428	26.0°	50.5°	26.60	1.50	6.00	22.90
9	0.324	1.7	2.0	1.176	"	"	26.30	1.80	7.20	21.60
10	0.322	2.7	3.5	1.296	"	"	25.90	2.20	8.80	20.20
11	0.328	4.1	5.0	1.219	"	"	25.40	2.70	10.80	18.30
12	0.309	5.5	7.0	1.273	"	"	24.70	3.40	13.60	16.20
13	0.339	6.6	8.0	1.212	"	"	24.35	3.75	15.00	14.75
14	0.346	7.8	9.5	1.218	"	"	23.80	4.30	17.20	13.0
15	0.364	9.3	11.0	1.183	"	"	23.30	4.80	19.20	11.00
16	0.376	10.5	12.5	1.190	"	"	22.80	5.30	21.20	9.30
17	0.417	12.1	14.0	1.157	"	"	22.30	5.80	23.20	7.20
18	0.438	13.1	15.5	1.183	"	"	21.80	6.30	25.20	5.70
19	0.588	14.8	17.0	1.149	"	"	21.20	6.90	27.60	3.40
20	0.600	15.4	18.5	1.201	"	"	20.90	7.20	28.80	2.50
21	0.909	16.5	20.0	1.212	"	"	20.60	7.50	30.00	1.10

TABLE 4.13.

TWISTING AT CONSTANT TENSION - 0.6 CM. WIDE STRIP - 25 CM. LENGTH

TWISTING TENSION - 5 GMS. POSITION OF TWISTING JAW 3.0CMS

Number of Turns Introduced	Turns /CM in the Twisted Part	Length of Strip wrapped along the Axis of Yarn Cms.	Number of Turns in the Wrapped Part	Turns/CM in the Wrapped Part	Helix Angle in the Twisted Part	Helix Angle in the Wrapped Part Along the Axis	Reading on the Scale As Length Contracts Cms.	Contraction Cms.	% Contraction	Length of Strip in Twisted Part Cms.
0	0	0	0	0	0	0	27.90	0	0	24.90
1	0.040	-	-	-	4.0°	-	27.85	0.05	0.20	24.85
2	0.81	-	-	-	7.0°	-	27.80	0.10	0.40	24.80
3	0.121	-	-	-	10.5°	-	27.70	0.20	0.80	24.70
4	0.163	-	-	-	15.0°	-	27.60	0.30	1.20	24.60
5	0.204	-	-	-	20.0°	-	27.45	0.45	1.80	24.45
6	0.248	-	-	-	23.5°	-	27.20	0.70	2.80	24.20
7	0.292	-	-	-	26.5°	-	27.00	0.90	3.60	24.00
8	0.311	0.7	1.0	1.428	25.5°	53.0°	26.20	1.70	6.80	22.50
9	0.316	2.1	2.5	1.190	"	"	25.65	2.25	9.00	20.55
10	0.316	3.1	4.0	1.290	"	"	25.10	2.80	11.20	19.00
11	0.324	4.3	5.5	1.279	"	"	24.25	3.65	14.60	16.95
12	0.329	5.4	7.0	1.296	"	"	23.60	4.30	17.20	15.20
13	0.341	6.6	8.5	1.288	"	"	22.80	5.10	20.40	13.20
14	0.354	7.8	10.0	1.282	"	"	22.10	5.90	23.60	11.30
15	0.364	8.9	11.5	1.292	"	"	21.50	6.40	25.60	9.60
16	0.411	10.4	13.0	1.250	"	"	20.70	7.20	28.80	7.30
17	0.454	11.5	14.5	1.261	"	"	20.00	7.90	31.60	5.50
18	0.588	12.8	16.0	1.250	"	"	19.20	8.70	34.80	3.40
19	0.714	13.7	17.5	1.277	"	"	18.80	9.10	36.40	2.10

TABLE 4.14.

TWISTING AT CONSTANT TENSION - 0.5 CM. WIDE STRIP - 25 CM LENGTH

TWISTING TENSION - 10 GMS   POSITION OF TWISTING JAW - 3.0 CMS

Number of turns introduced	Turns /CM. in the twisted part	Length of strip wrapped along the Axis of Yarn Cms.	Number of turns in the wrapped part	Turns /CM. in the wrapped part	Helix Angle in the Twisted part	Helix Angle in the wrapped part along the axis	Reading on the scale as length contracts Cms.	Contraction Cms.	% Contraction	Length of strip in Twisted Part Cms.
0	0	0	0	0	0	0	28.0	0	0	25.0
1	0.04	-	-	-	2°	-	28.0		0	25.0
2	0.08	-	-	-	5°	-	28.0	0	0	25.0
3	0.120	-	-	-	9°	-	28.0	0	0	25.0
4	0.150	-	-	-	13°	-	27.9	0.1	0.40	24.90
55	0.202	-	-	-	16°	-	27.8	0.2	0.80	24.80
6	0.243	-	-	-	13°	-	27.7	0.3	1.20	24.70
7	0.287	-	-	-	22.5°	-	27.4	0.6	2.40	24.40
8	0.330	-	-	-	27.5°	-	27.2	0.8	3.20	24.20
9	0.375	-	-	-	31.0°	-	27.0	1.00	4.00	24.00
10	0.380	1.0	1.5	1.50	30.0°	51.0°	26.35	1.65	6.60	22.35
11	0.405	2.0	3.5	1.25	"	"	26.00	2.00	8.00	21.00
12	0.391	3.3	4.5	1.36	"	"	25.50	2.50	10.00	19.20
13	0.419	4.3	5.5	1.28	"	"	25.20	2.80	11.20	17.90
14	0.396	5.4	7.5	1.39	"	"	24.80	3.20	12.80	16.40
15	0.376	6.4	9.5	1.48	"	"	24.40	3.60	14.40	15.00
16	0.404	7.5	10.5	1.40	"	"	24.10	3.90	14.60	13.60
17	0.378	8.7	12.5	1.44	"	"	23.60	4.40	17.60	11.90
18	0.379	9.7	14.0	1.44	"	"	23.25	4.75	19.00	10.55
19	0.398	11.0	15.5	1.41	"	"	22.80	5.20	20.80	8.80
20	0.411	12.1	17.0	1.41	"	"	22.40	5.60	22.40	7.30
21	0.446	13.4	18.5	1.38	"	"	22.00	6.00	24.00	5.60
22	0.395	14.7	20.5	1.39	"	"	21.50	6.50	26.00	3.80
23	0.344	15.2	22.0	1.45	"	"	21.10	6.90	27.60	2.90

TABLE 4.15

TWISTING AT CONSTANT TENSION - 0.5 CMS. WIDE STRIP - 25 CMS.LENGTH

TWISTING TENSION - 5 GMS. POSITION OF TWISTING JAW - 3.0 CMS.

Number of Turns Introduced	Turns/CM in the Twisted Part	Length of Strip Wrapped along the Axis of Yarn Cms.	Number of Turns in the wrapped Part	Turns/CM in the Wrapped Part	Helix Angle in the Twisted Part	Helix Angle in the Wrapped Part Along the Axis	Reading on Scale as Length Contracts Cms.	Contraction Cms.	% Contraction	Length of Strip in Twisted Part Cms.
0	0	0	0	0	0	0	28.0	0	0	25.00
1	0.040	-	-	-	2°	-	28.0	0	0	25.00
2	0.080	-	-	-	5.5°	-	28.0	0	0	25.00
3	0.120	-	-	-	9.0°	-	27.95	0.05	0.20	24.95
4	0.161	-	-	-	12.5°	-	27.85	0.15	0.60	24.85
5	0.202	-	-	-	16.0°	-	27.75	0.25	1.00	24.75
6	0.244	-	-	-	18.5°	-	27.60	0.40	1.60	24.60
7	0.286	-	-	-	22.0°	-	27.45	0.55	2.20	24.45
8	0.330	-	-	-	24.5°	-	27.25	0.75	3.00	24.25
9	0.373	-	-	-	28.5°	-	27.10	0.90	3.60	24.10
10	0.414	0.4	0.5	1.250	28.0°	54.0°	26.35	1.65	6.60	22.95
11	0.429	1.7	2.0	1.176	"	"	25.65	2.35	9.40	20.95
12	0.431	2.4	3.5	1.458	"	"	25.10	2.90	11.60	19.70
13	0.435	3.3	5.0	1.515	"	"	24.70	3.30	13.20	18.40
14	0.454	4.6	6.5	1.413	"	"	24.10	3.90	15.60	16.50
15	0.408	5.8	9.0	1.552	"	"	23.50	4.50	18.00	14.70
16	0.479	6.3	9.5	1.508	"	"	22.85	5.15	20.60	13.55
o7	0.466	7.6	11.5	1.513	"	"	22.40	5.60	22.40	11.80
18	0.483	8.6	13.0	1.512	"	"	21.95	6.05	24.20	10.35
19	0.529	10.0	14.5	1.450	"	"	21.50	6.50	26.00	8.50
20	0.515	10.9	16.5	1.514	"	"	20.70	7.30	29.20	6.80
21	0.550	11.8	18.0	1.525	"	"	20.25	7.75	31.00	5.45
22	0.540	12.9	20.0	1.550	"	"	19.60	8.40	33.60	3.70
23	0.833	13.8	21.0	1.522	"	"	19.20	8.80	35.20	2.40

TABLE 4.16.

TWISTING AT CONSTANT TENSION - 0.4 CMS. WIDE -.25 CMS LENGTH

TWISTING TENSION - 10 GMS   POSITION OF TWISTING JAW - 3.0 CMS.

Number of Turns Introduced	Turns/CM in the Twisted Part	Length of Strip wrapped along the Axis of Yarn	Number of Turns in the Wrapped Part	Turns/CM in the Wrapped Part	Helix Angle in the Twisted Part	Helix Angle in the Wrapped Part Along the Axis	Reading on the Scale At Length Contracts	Contraction	% Contraction	Length of Strip In Twisted Part
0	0	0	0	0	0	0	28.05	0	00	25.05
1	0.040	-	-		1.5°	-	28.05	0	0	25.05
2	0.080	-	-		4.0°	-	28.05	0	0	25.05
3	0.120	-	-		7.0°	-	28.05	0	0	25.05
4	0.160	-	-		9.0°	-	28.05	0	0	25.05
5	0.200	-	-		12.0°	-	28.05	0	0	25.05
6	0.240	-	-		14.5°	-	28.05	0	0	25.05
7	0.280	-	-		16.0°	-	28.00	0.05	0.20	25.00
8	0.321	-	-		19.0°	-	27.90	0.15	0.60	24.90
9	0.362	-	-		21.5°	-	27.85	0.20	0.80	24.85
10	0.405	-	-		23.0°	-	27.65	0.40	1.60	24.65
11	0.449	-	-		25.5°	-	27.50	0.55	2.20	24.50
12	0.489	-	-		28.0°	-	27.50	0.55	2.20	24.50
13	0.533	-	-		30.5°	-	27.40	0.65	2.60	24.40
14	0.576	-	-		32.0°	-	27.30	0.75	3.00	24.30
15	0.621	-	-		33.0°	-	27.15	0.90	3.60	24.15
16	0.665	-	-		34.0°	-	27.05	1.00	4.00	24.05
17	0.695	0.7	1.0	1.428	34.0°	52.5°	26.70	1.35	5.40	23.00
18	0.711	1.6	2.5	1.564	"	"	26.40	1.65	6.60	21.80
19	0.721	2.4	4.0	1.668	"	"	26.20	1.85	7.40	20.80
20	0.705	3.1	6.0	1.935	"	"	25.95	2.10	8.40	19.85
21	0.710	4.2	8.0	1.905	"	"	25.50	2.55	10.20	18.30
22	0.718	4.8	9.5	1.979	"	"	25.20	2.85	11.40	17.40
23	0.694	5.9	12.0	2.034	"	"	24.75	3.30	13.20	15.85
24	0.707	6.8	13.5	1.985	"	"	24.65	3.40	13.60	14.85

TABLE 4.16 (Continued)

Number of Turns Introduced	Turns/CM in the Twisted Part	Length of Strip Wrapped along the Axis of Yarn Cms.	Number of Turns in the Wrapped Part	Turns/CM in the Wrapped Part	Helix Angle in the Twisted Part	Helix Angle in the Wrapped Part Along the Axis	Reading on the Scale As Length Contracts Cms.	Contraction Cms.	% Contraction	Length of Strip in Twisted Part Cms.
25	0.727	7.6	15.0	1.974	34.0°	52.5°	24.35	3.70	14.80	13.75
26	0.603	9.2	19.0	2.065	"	"	23.80	4.25	17.00	11.60
27	0.591	10.3	21.0	2.039	"	"	23.45	4.60	18.40	10.15
28	0.598	11.0	22.5	2.045	"	"	23.20	4.85	19.40	9.20
29	0.641	12.0	24.0	2.000	"	"	22.80	5.25	21.00	7.80
30	0.534	13.0	26.5	2.038	"	"	22.55	5.50	22.00	6.55
31	0.603	13.6	27.5	2.106	"	"	22.40	5.65	22.60	5.80
32	0.454	14.7	30.0	2.041	"	"	22.10	5.95	23.80	4.40
33	0.484	15.6	31.5	2.019	"	"	21.70	6.35	25.40	3.10
34	0.303	16.7	33.5	2.006	"	"	21.35	6.70	26.80	1.65

TABLE 4.17.

TWISTING AT CONSTANT TENSION - 0.4 Cm. WIDE STRIP - 25 Cms. LWNGTH

TWISTING TENSION - 5 GMS. POSITION OF TWISTING JAW - 3.0 CMS.

Number of Turns Introduced	Turns/cm in the Twisted Part	Length of strip wrapped along the axis of yarn	Number of Turns in the Wrapped Part	Turns/CM in the Wrapped Part	Helix Angle in the Twisted Part	Helix Angle in the Wrapped Part Along the Axis	Reading on the Scale as Length Contracts	Contraction	% Contraction	Length of Strip in Twisted Part
0	0	0	0	0	0	0	28.05	0	0	25.05
1	0.040	-	-	-	1.0°	-	28.05	0	0	25.05
2	0.080	-	-	-	3.5°	-	28.05	0	0	25.05
3	0.120	-	-	-	6.5°	-	28.05	0	0	25.05
4	0.160	-	-	-	7.5°	-	28.05	0	0	25.05
5	0.200	-	-	-	10.5°	-	28.05	0	0	25.05
6	0.240	-	-	-	12.5°	-	27.95	0.10	0.40	24.95
7	0.281	-	-	-	15.0°	-	27.90	0.15	0.60	24.90
8	0.322	-	-	-	16.0°	-	27.85	0.20	0.80	24.85
9	0.362	-	-	-	18.5°	-	27.85	0.20	0.80	24.85
10	0.406	-	-	-	21.0°	-	27.70	0.35	1.40	24.70
11	0.447	-	-	-	22.0°	-	27.60	0.45	1.80	24.60
12	0.490	-	-	-	24.0°	-	27.50	0.55	2.20	24.50
13	0.531	-	-	-	26.5°	-	27.50	0.55	2.20	24.50
14	0.577	-	-	-	28.0°	-	27.25	0.80	3.20	24.25
15	0.623	-	-	-	31.0°	-	27.10	0.90	3.60	24.10
16	0.669	-	-	-	33.0°	56.0°	26.90	1.15	4.60	23.90
17	0.705	0.60	1.0	1.667	"	"	26.30	1.75	7.00	22.70
18	0.690	1.9	3.5	1.842	"	"	25.90	2.15	8.60	21.00
19	0.707	2.6	5.0	1.923	"	"	25.40	2.65	10.60	19.80
20	0.710	3.8	7.5	1.974	"	"	24.40	3.65	15.60	17.60
21	0.706	4.3	9.0	2.093	"	"	24.30	3.75	15.00	17.00
22	0.734	4.8	10.0	2.083	"	"	24.15	3.90	15.60	16.35
23	0.714	5.9	12.5	2.119	"	"	23.60	4.45	17.80	14.70
24	0.745	6.3	13.5	2.143	"	"	23.40	4.65	18.60	14.10

Table 4.17 (Continued)

Number of Turns Introduced	Turns/cm in the Twisted Part	Length of Strip Wrapped Along the Axis of Yarn cms	Number of Turns in the Wrapped Part	Turns/CM in the Wrapped Part	Helix Angle in the Twisted Part	Helix Angle in the Wrapped Part	Along the Axis Reading on the Scale as Length Contracts cms	Contraction cms	% Contraction	Length of Strip in Twisted Part cms
25	0.781	7.2	15.0	2.083	33.0°	56.0°	23.00	5.05	20.20	12.80
26	0.812	7.9	16.5	2.089	"	"	22.60	5.45	21.80	11.70
27	0.800	9.0	19.0	2.111	"	"	22.00	6.05	24.20	10.00
28	0.751	9.9	21.5	2.172	"	"	21.55	6.50	26.20	8.65
29	0.724	11.0	24.0	2.182	"	"	20.90	7.15	28.60	6.9
30	0.789	11.8	25.5	2.161	"	"	20.50	7.55	30.20	5.7
31	0.918	12.3	26.5	2.155	"	"	20.20	7.85	31.40	4.9
32	0.968	13.3	29.0	2.180	"	"	19.40	8.65	34.60	3.1
33	1.250	14.4	31.0	2.153	"	"	19.00	9.05	36.20	1.6

TABLE 4.18.

TWISTING AT CONSTANT TENSION
LENGTH OF RIBBON - 25 CMS.

Tension in Gms.	Ribbon Width in Cms.	Helix Angle in the Twisted Part	Helix Angle in the Wrapped Part	Number of Turns at which Wrapping Occurs	Turns/Cm
2.5	0.25	36.5°	45.0°	27.9	1.116
	0.40	29.5°	57.0°	15.8	0.632
	0.50	25.5°	56.0°	9.0	0.360
	0.60	24.5°	55.0°	6.6	0.264
	0.80	23.0°	54.0°	5.2	0.208
	1.00	17.0°	52.5°	3.7	0.148
5.0	0.25	40.0°	59.0°	29.6	1.184
	0.40	33.0°	56.0°	16.0	0.640
	0.50	28.0°	54.0°	9.7	0.388
	0.60	25.5°	53.0°	7.0	0.280
	0.80	24.0°	52.5°	5.5	0.220
	1.00	20.0°	50.0°	3.7	0.148
7.5	0.25	42.0°	58.0°	31.6	1.264
	0.40	33.0°	53.5°	17.0	0.680
	0.50	29.0°	53.0°	10.2	0.408
	0.60	25.5°	51.5°	7.4	0.296
	0.80	24.5°	50.0°	5.7	0.228
	1.00	21.0°	46.0°	3.8	0.154
10.0	0.25	43.5°	56.0°	33.3	1.332
	0.40	34.0°	52.5°	17.6	0.704
	0.50	30.0°	51.0°	10.5	0.420
	0.60	26.0°	50.5°	7.7	0.308
	0.80	25.0°	48.0°	5.8	0.232
	1.00	22.0°	45.5°	4.0	0.160

Table 4.18 (Continued)

Tension in Gms.	Ribbon Width in Cms.	Helix Angle in the		Number of Turns at which Wrapping Occurs	Turns/Cm
		Twisted Part	Wrapped Part		
12.5	0.25	46.0°	57.5°	34.6	1.384
	0.40	34.0°	50.0°	18.2	0.728
	0.50	30.0°	49.5°	10.8	0.432
	0.60	25.5°	47.5°	8.0	0.320
	0.80	26.5°	46.5°	5.9	0.236
	1.00	23.5°	44.5°	4.1	0.164
15.0	0.25	48.0°	58.0°	36.6	1.464
	0.40	38.5°	49.0°	18.8	0.752
	0.50	33.0°	48.0°	11.2	0.448
	0.60	30.0°	46.5°	7.9	0.316
	0.80	28.0°	45.5°	6 1	0.244
	1.00	24.5°	42.5°	4.1	0.164

TABLE 4.19

MEASUREMENT OF THE THICKNESS OF RUBBER STRIP BY R & B TESTER

Least Count of the instrument = 0.01 mm.

S.No.	Reading on the Scale (Divs.)	Thickness in mm.
1	90	0.90
2	89	0.89
3	90	0.90
4	88	0.88
5	92	0.92
6	90	0.90
7	90	0.90
8	88	0.88
9	91	0.91
10	89	0.89
11	91	0.91
12	91	0.91
13	90	0.90
14	90	0.90
15	89	0.89
16	89	0.89
17	88	0.88
18	89	0.89
19	91	0.91
20	91	0.91
MEAN	89.8	0.898 mm.

Weight of rubber materials in gms/cm^2

Rubber strips of size 3" x 3.5" were cut and weighed in Mettler's Yarn Balance (Mettler - is the trade name of the Balance made by a Swiss firm).

TABLE 4.20

S. No.	Weight of rubber strips in gms.
1	4.9082
2	4.5612
3	4.7348
4	4.8124
5	4.7320
6	5.0196
7	4.8482
8	4.8896
9	4.8876
10	4.7285
11	4.7104
12	4.6100
13	4.9743
14	4.6874
15	4.6000
MEAN WEIGHT	4.7803 gms.

$$\therefore \quad gms/cm^2 = \frac{4.7803}{3 \times 3.5 \times (2.54)^2}$$

$$= \frac{4.7803}{67.7418}$$

$$= \underline{\underline{0.0706}}$$

TABLE 4.21

DETERMINATION OF BENDING MODULUS OF RUBBER STRIP BY DIRECT MEASUREMENT OF BENDING LENGTH BY SHIRLEY STIFFNESS TESTER. BENDING LENGTH SAMPLES ARE 1 inch wide.

	ONE END		OTHER END	
	Top Face Up	Reversed Face Up	Top Face Up	Reversed Face Up
	2.25 cms.	1.95 cms.	2.20 cms.	1.95 cms.
	2.30 "	1.85 "	2.45 "	1.70 "
	2.20 "	2.00 "	2.15 "	2.00 "
	2.20 "	2.00 "	2.15 "	1.80 "
	2.40 "	1.80 "	2.30 "	1.75 "
MEAN	2.27 "	1.95 "	2.25 "	1.85 "

MEAN OF ALL = 2.08 cms.

Calculation of Bending Modulus

$$\text{Bending Modulus} = q = \frac{12\ w\ c^3}{d^3}$$

where w = weight of material in gms/cm^2

c = bending length in cm.

d = thickness of material in cm.

$$\therefore\ q = \frac{12 \times 0.0706 \times (2.08)^3}{(0.09)^3}$$

$$= 10307.9012\ \text{gms/cm}^2.$$

TABLE 4.22

DETERMINATION OF YOUNG'S MODULUS OR BENDING MODULUS OF 1 cm. WIDE RUBBER STRIPS BY INSTRON.

Length of specimen	=	15 cm.
Full scale deflection	=	100 gms.
Chart speed	=	1 cm/min.
Jaw speed	=	1 cm/min.
∴ Magnification	=	1

S.No.	LOAD FOR CALCULATING INITIAL MODULUS	EXTENSION AT THIS LOAD
1	100 gms.	1.6 cms.
2	"	1.4 cms.
3	"	1.75 cms.
4	"	1.50 cms.
5	"	1.35 cms.
MEAN	100 gms.	1.52 cms.

Calculation

$$\text{Initial Modulus} = \frac{\text{Stress}}{\text{Strain}} \times \text{magnification}$$

$$= \frac{\text{Load/Area of Cross-Section}}{\text{Extension/Gauge Length}} \times \frac{\text{Chart Speed}}{\text{Cross-Head Speed}}$$

$$= \frac{100/0.09 \times 1}{1.52/15} \times \frac{1}{1}$$

$$= \frac{1500}{0.1368} =$$

$$= 10964.9123 \text{ gms/cm}^2$$

TABLE 4.23

DETERMINATION OF TORSIONAL MODULUS OF RUBBER STRIP

Length of strip	=	53.7 cms.
Width of strip	=	1.0 cms.
Thickness of strip	=	0.09 cms.
Diameter of the pulley	=	5.56 cms.

LOAD APPLIED	NUMBER OF TURNS INTRODUCED	
	TWISTED IN ONE DIRECTION	TWISTED IN OPPOSITE DIRECTION
1 gm.	4.5	4.5
"	4.5	4.5
"	4.5	4.5
"	4.5	4.5
	4.5	4.5

Calculations

$$\frac{2}{3} \pi \eta bd^3 \quad \frac{n_1}{\ell_1} = M g R^1$$

$$\therefore \quad \eta = \frac{3}{2} \times \frac{7}{22} \times \frac{1}{1 \times (0.09)^3} \times \frac{25}{4.5} \times 1 \times \frac{5.56}{2} \times g$$

$$= 21719.29 \times g \text{ dynes/cm}^2$$

$$= 21719.29 \text{ gms/cm}^2$$

where g is the gravitational force in C.G.S. Units.

CHAPTER V

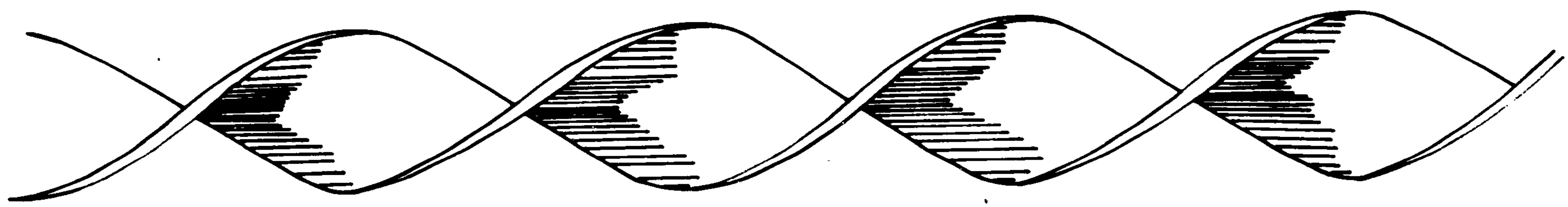
Geometry and Mechanics of Twisting of Rubber Strips

5.1 Geometry of twisted form

The geometrical relations for the twisted form of ribbon can be arrived at by considering it to be of the same type as twisting of a thin metal blade. Figure 5.1 shows the general form of twisted structure. In Figure 5.2(a) it is seen that the lower end of a cylinder whose top end is fixed has been rotated through a certain angle in the direction of the arrow.

Let OO^1 be the axis of the cylinder, and A is any point on the surface of the cylinder at the free end. After twisting, the point A acquires a new position B. Thus the point A has rotated through certain angle with respect to a similar point A^1 on the fixed end of the cylinder. The angle AA^1B is the helix angle. Now if the cylinder is considered to be made up of a number of thin parallel plates then on twisting the cylinder the plates will also be rotated in the similar fashion. If we consider a point C on a plate at the centre of the cylinder then on twisting this point C would have moved to point C^1 and the angle CDC^1 will be equal to the angle AA^1B.

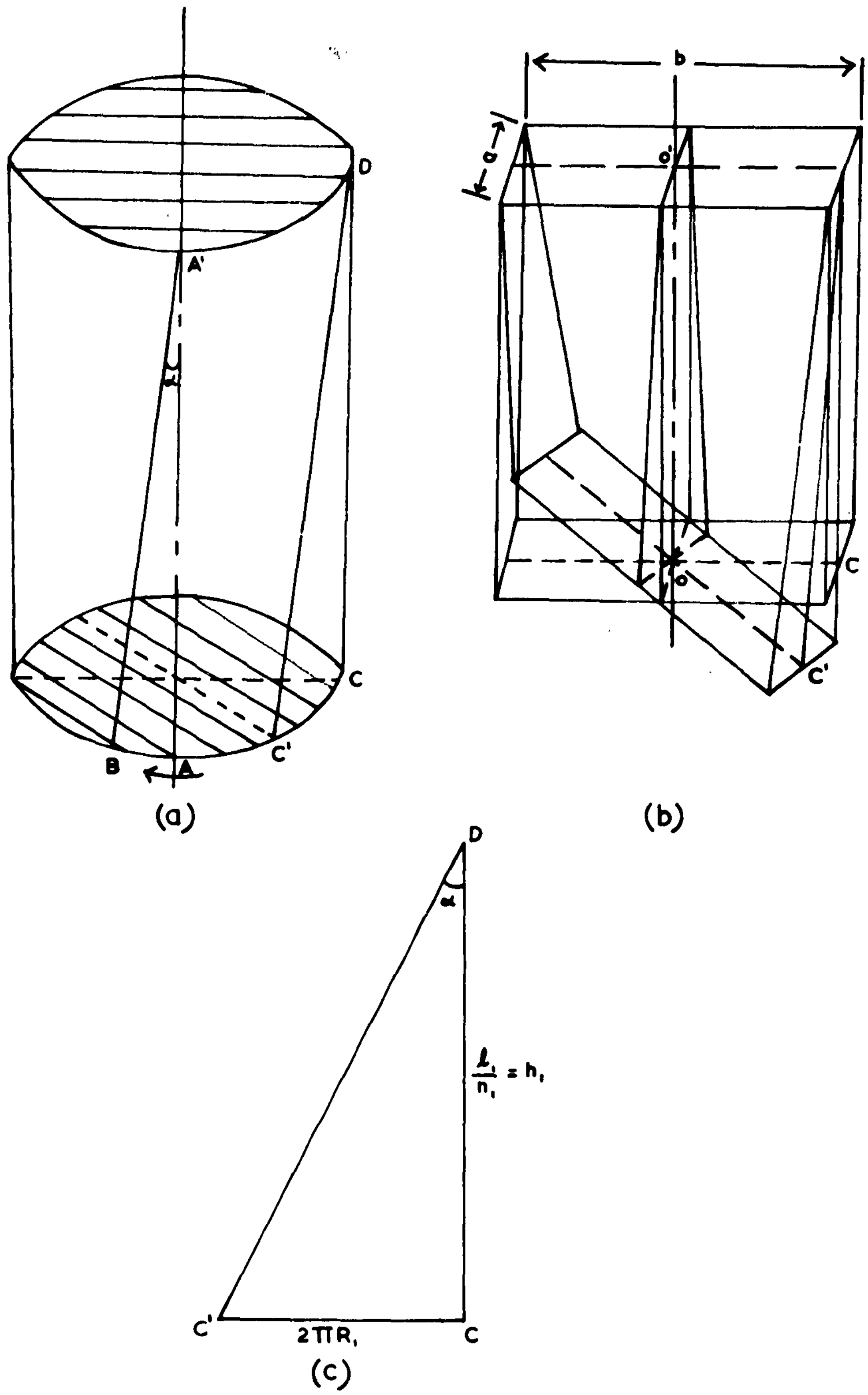
In Figure 5.2(b) twisting of the plate, which was in the centre of the cylinder, has been shown separately. Figure 5.2(c) has been drawn to show the movement of point C to C^1 and D is the point on the fixed end of the plate.



TWISTED FORM.

FIG. 5.1.

FIG. 5. 2.



In order to work out the geometry of the twisted part, let

ℓ_1 be the length of strip in twisted part

n_1 be the number of turns in twisted part

R_1 be the radius of the cylinder of which it is a part

α_1 be the helix angle

b be the width of the strip.

Assuming that (a) there is very little deformation in the twisted part, and (b) that the length along strip is equal to the length along the axis, i.e. there is negligible contraction in length, then from Figure 5.2(c):

$$\text{Length in one turn} = CD = \frac{\ell_1}{n_1} = h_1 \text{ (say)}$$

$$C^1D = 2\pi R_1 \operatorname{cosec}\alpha_1$$

$$\text{and } \tan\alpha_1 = \frac{2\pi R_1}{\ell_1/n_1} \qquad \text{.............} \quad (5.1)$$

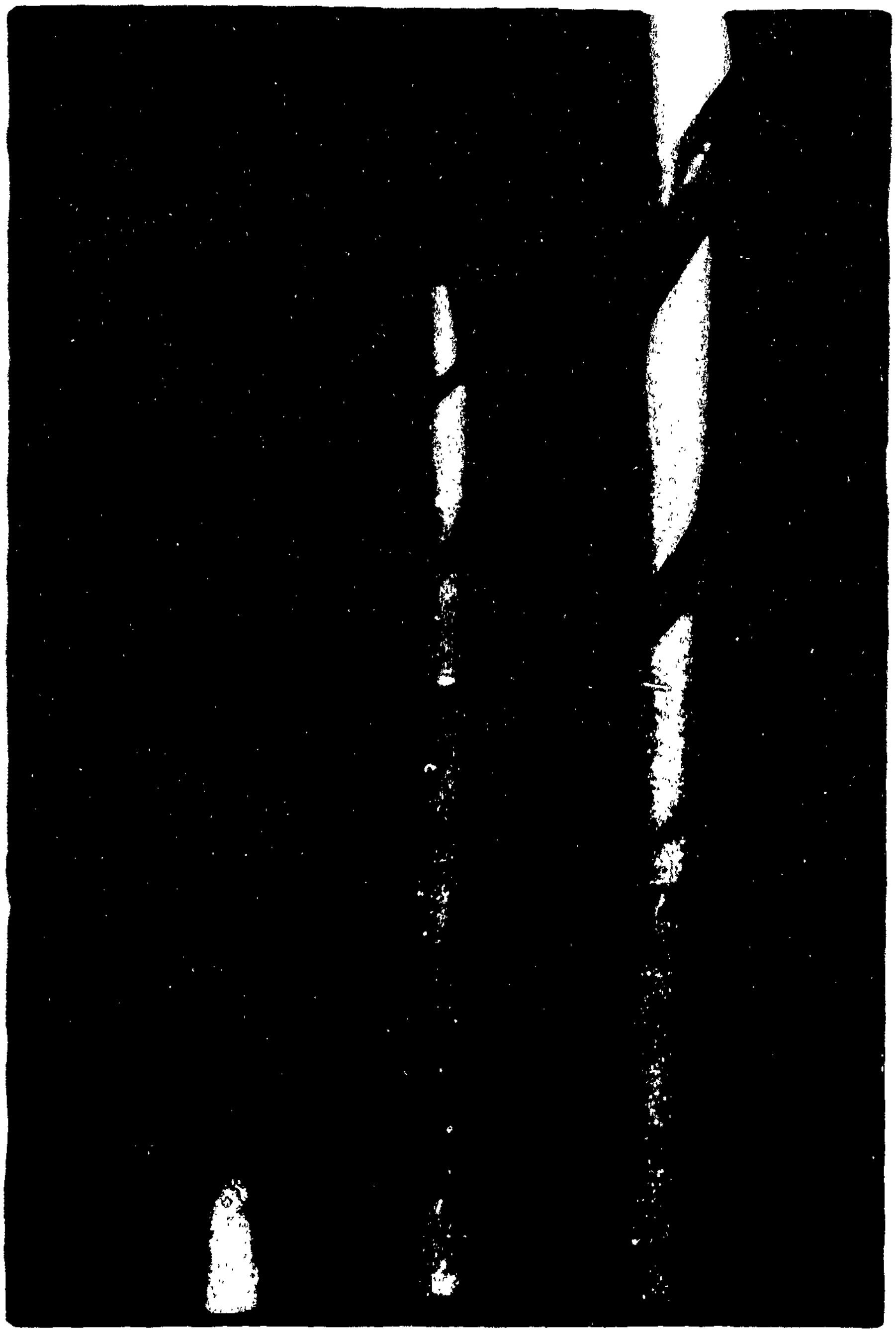
From our assumption (a)

$$R_1 = b/2$$

$$\text{therefore } C^1D = \pi b \operatorname{cosec}\alpha_1 \qquad \text{.................} \quad (5.2)$$

$$\text{and } \tan\alpha_1 = \frac{\pi b\,\ell}{\ell_1/n_1} \qquad \text{.............} \quad (5.3)$$

$$\text{also } C^1D = \left\{\pi^2 b^2 + \left(\frac{\ell_1}{n_1}\right)^2\right\}^{\frac{1}{2}} \qquad \text{.............} \quad (5.4)$$



a b c

FIG. 5.3.

5.2 Geometry of wrapped structure

The form of wrapped structure may be visualised by considering a strip of paper rolled up so that its axis is in one plane as illustrated in Fig. 5.3(a). This is a wrapped structure with a complete overlap of successive turns. If the ends of the paper are now pulled apart, a wrapped structure with only partial overlap is obtained, as in Fig. 5.3(b). Finally, on further extending the chain, a gap between turns will form as in Fig. 5.3(c). Thus there are three types of wrapped structure.

(a) Wrapped structure with overlapping of turns, Fig. 5.4(c). In practice with a strip of rubber of finite thickness the overlap would not be able to take place, and the turns would join together under the condition in which they are trying to overlap.

(b) Wrapped structure with no overlap and no gap, Fig. 5.4(b) i.e. a structure which is just jammed.

(c) Wrapped structure with a gap, Fig. 5.4 (a).

In order to work out the detailed geometry of the wrapped part, let

the length in the wrapped part along the strip = ℓ_2

the length in the wrapped part along the axis = L_2 cms

the helix angle be = α_2

the number of turns in the wrapped part = n_2

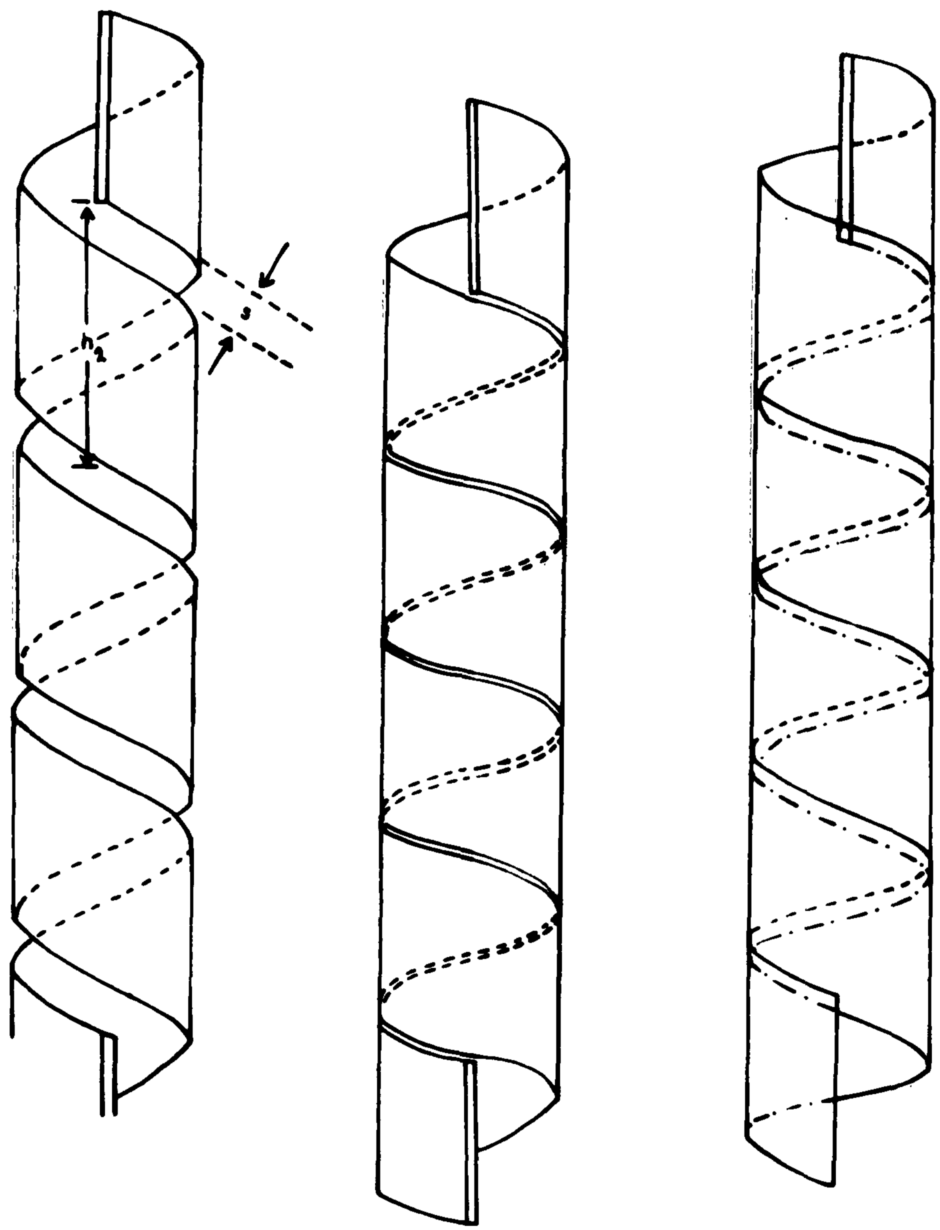
and the radius of the cylinder = R_2

Let S be the separation between the strips perpendicular to the length of the strip as shown in figure. If there is overlap S will be negative.

Fig. 5.5 (a) and (b) has been obtained by cutting the wrapped structure and opening it out into a plane. We see that

FIG. 5.4.

FORMS IN WRAPPED STRUCTURE.



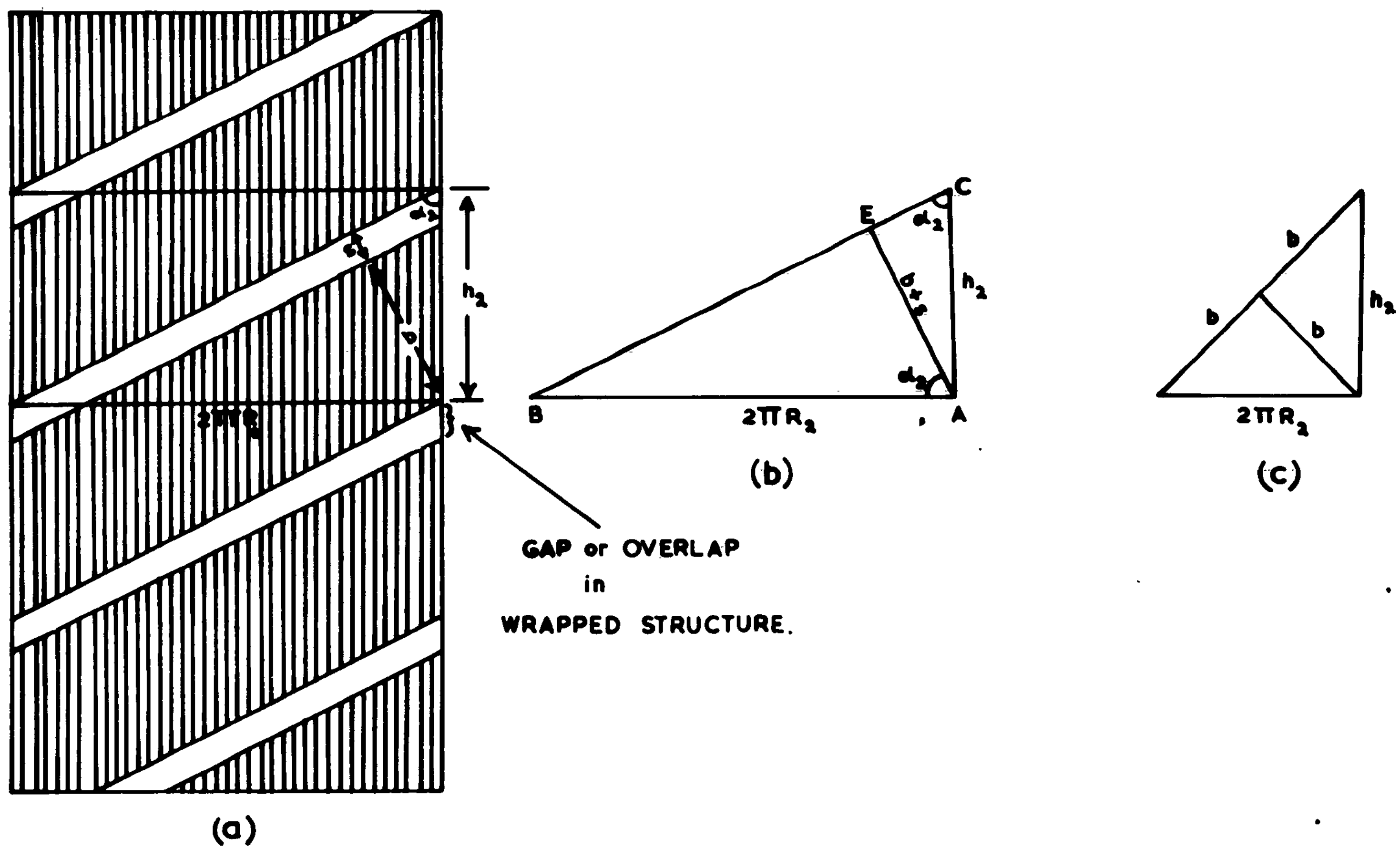
WITH GAP. (a)

JUST JAMMED. (b)

WITH OVERLAP. (c)

FIG. 5. 5.

GEOMETRY OF WRAPPED STRUCTURE.



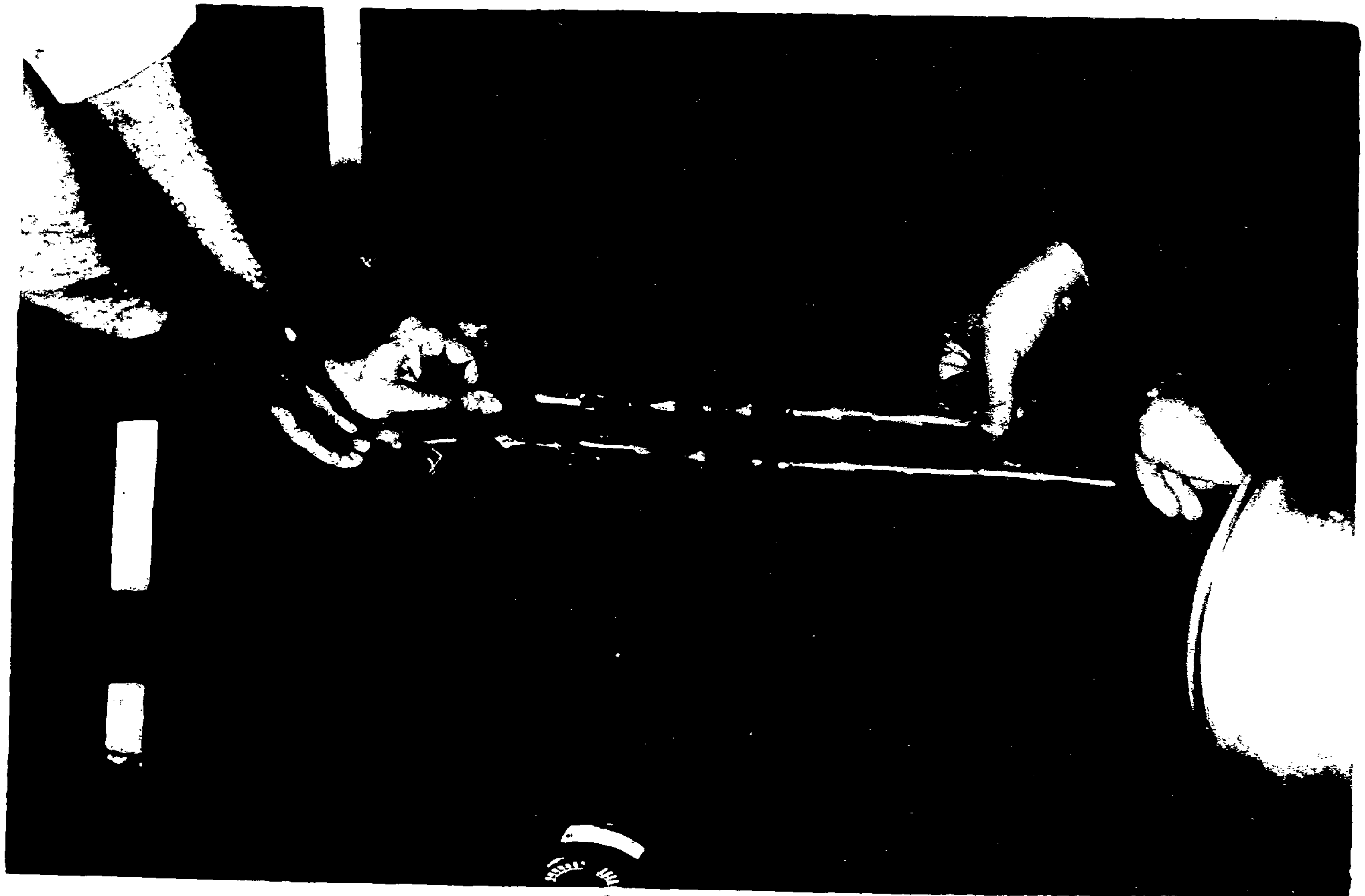


FIG. 5.6.

$$\tan\alpha_2 = \frac{2\pi R_2}{h_2} \qquad \ldots\ldots\ldots\ldots (5.5)$$

$$\frac{b+S}{h_2} = \sin\alpha_2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (5.6)$$

$$\frac{h_2}{\ell_2/n_2} = \cos\alpha_2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (5.7)$$

$$\frac{2\pi R_2}{\ell_2/n_2} = \sin\alpha_2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (5.8)$$

$$\frac{b+S}{2\pi R_2} = \cos\alpha_2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (5.9)$$

from (8) and (9) we get $\quad \dfrac{(b+S)^2}{4\pi^2 R_2^2} = \cos^2\alpha_2$

$$\frac{4\pi^2 R_2^2}{(\ell_2/n_2)^2} = \sin^2\alpha_2$$

$$\therefore \quad \frac{4\pi^2 R_2^2}{(\ell_2/n_2)^2} = 1 - \frac{(b+S)^2}{4\pi^2 R_2^2} \qquad \ldots\ldots\ldots\ldots\ldots (5.10)$$

or $\quad 16\pi^4 R_2^4 = \left(\dfrac{\ell_2}{n_2}\right)^2 \left\{4\pi^2 R_2^2 - (b+S)^2\right\}$

or $\quad 16\pi^4 R_2^4 - 4\pi^2 \dfrac{\ell_2^2}{n_2^2} R_2^2 + (b+S)^2 \dfrac{\ell_2^2}{n_2^2} = 0.$

$$\therefore \quad R_2^{\,2} = \frac{4\pi^2 \dfrac{\ell_2^{\,2}}{n_2^{\,2}} \pm \sqrt{16\pi^4 \left(\dfrac{\ell_2^{\,2}}{n_2^{\,2}}\right)^2 - 64\pi^4 (b+S)^2 . \dfrac{\ell_2^{\,2}}{n_2^{\,2}}}}{2 .\ 16\pi^4}$$

$$= \frac{4\pi^2 \left[\dfrac{\ell_2^{\,2}}{n_2^{\,2}} \pm \dfrac{\ell_2}{n_2}\sqrt{\dfrac{\ell_2^{\,2}}{n_2^{\,2}} - 4(b+S)^2}\right]}{32\,\pi^4}$$

$$\text{or } R_2^{\,2} = \frac{\dfrac{\ell_2^{\,2}}{n_2^{\,2}} \pm \dfrac{\ell_2}{n_2}\sqrt{\dfrac{\ell_2^{\,2}}{n_2^{\,2}} - 4\,(b+S)^2}}{8\,\pi^2}$$

$$\text{or } R_2^{\,2} = \frac{\ell_2^{\,2} \pm \ell_2 \sqrt{\ell_2^{\,2} - 4\,(b+S)^2 n_2^{\,2}}}{8\pi^2\, n_2^{\,2}} \qquad \text{.........} (5.11)$$

Also from Fig. 5.5(b) we find that

$$h_2 = (b + S) \operatorname{cosec} \alpha_2 \qquad \text{..........} (5.12)$$

Equation (5.12) gives the general formula for the length of one turn of the structure along the axis and equation (5.11) is the general equation for the radius of the wrapped structure. S can assume positive or negative values or it can be zero. When S has positive values we have wrapped structure with gap, but when S is negative we obtain the wrapped structure with overlapping. The zero value of S corresponds to the jammed structure, but the general form of the equation (5.11) and (5.12) will be used initially in dealing with the mechanics of the wrapped structure.

Equation (5.11) has two solutions because it is in general possible, as illustrated in Fig. 5.6, to have two structures with the same length of strip, the same number of turns, and the same gap (or overlap) between the turns of the strip.

It is interesting to consider some special cases:

(i) Considering the positive sign only in equation (5.11)

$$R_2^2 = \frac{\ell_2^2 + \ell_2\sqrt{\ell_2^2 - 4(b + S)^2 n_2^2}}{8\pi^2 n_2^2} \quad \ldots (5.11a)$$

(ii) When $S = -b$, i.e. there is complete overlap

$$R_2^2 = \frac{\ell_2^2}{4\pi^2 n_2^2} \quad \ldots\ldots\ (5.11b)$$

This is correct since the strip is wrapped in one plane Fig. 5.3(a) and its circumference must equal ℓ_2/n_2 .

(iii) When S = 0, i.e. the structure is just jammed.

$$R_2^{\,2} = \frac{\ell_2^{\,2} \pm \ell_2 \quad \ell_2^{\,2} - 4\,b^2 n_2^{\,2}}{8\pi^2\, n_2^{\,2}} \qquad \text{..... (5.11c)}$$

Again two solutions are in general possible, corresponding to jamming, in tension or compression.

(iv) When S = 0 and $n_2 = \dfrac{\ell_2}{2b}$

$$R_2^{\,2} = \frac{\ell_2^{\,2}}{8\pi^2\, n_2^{\,2}} \qquad \text{...... (5.11d)}$$

In this case the two solutions merge into one. With this number of turns only one structure is possible without overlap. Since $\dfrac{\ell_2}{n_2} = 2b$, the geometry of the structure will be as shown in Fig. 5.5(c), with $\alpha_2 = 45^{\circ}$.

5.3 Mechanics of the twisted part

Let the width of the ribbon be b cms

thickness of the ribbon be d cms

length of the ribbon be ℓ cms.

* If the thicknes 'd' of the ribbon be very small compared to the width b, then the standard expression for the couple required to twist a thin blade, one end of which is fixed, can be used for our purpose.

* Experimental Elasticity - Searle - Page 68.

$\therefore$ for a rectangular strip of blade

The couple $G = \frac{1}{3}\,\eta\,\tau\; b.d^3$ dynes cm.

where η is the shear modulus

and τ' is the twist in radians/cm.

or $G = \frac{1}{3}\,\eta\,\frac{\theta}{l}\; b\, d^3$

where θ is the rotation of the free end in radians.

Applying this equation to the twisted part of the structure

we know that $\theta = 2\pi n_1$

and $l = l_1$

$\therefore$ Couple G becomes $= \frac{1}{3}\,\eta\,\frac{2\pi n_1}{l_1}\; b\, d^3$

$$= \frac{2}{3}\pi\,\eta\; b\, d^3\; \frac{n_1}{l_1} \text{ dynes cm.} \qquad \ldots\ldots (5.13)$$

Strain Energy

When a length of a bar is twisted through certain angle then work is done on it and this work done is stored in the body as energy. If the body is a perfectly elastic one, then this energy is released on removal of the forces causing this strain. Thus, if G be the couple applied to twist a body through an angle θ, then the work done

$$= \text{Strain Energy} = \tfrac{1}{2}\, G\, \theta$$

In our case of twisting a rectangular blade

$$\text{Strain Energy} = E_r = \tfrac{1}{2}\, G.\theta$$

$$= \tfrac{1}{2}.\ \tfrac{2}{3}\pi\,\eta\, b\, d^3\ \frac{n_1}{l_1}\ 2\ \ n_1$$

$$= \tfrac{2}{3}\,\pi^2\,\eta\, b\, d^3\ \frac{n_1^{\,2}}{l_1} \qquad \ldots\ldots\ldots (5.14)$$

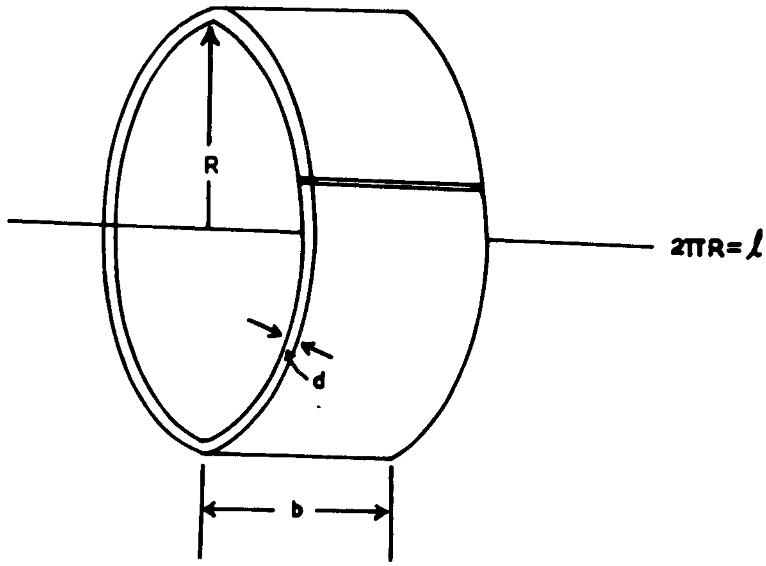
$$\text{or } E_r = \pi\, n_1\, G \qquad \ldots\ldots\ldots\ldots (5.15)$$

5.4 Mechanics of Wrapped Part

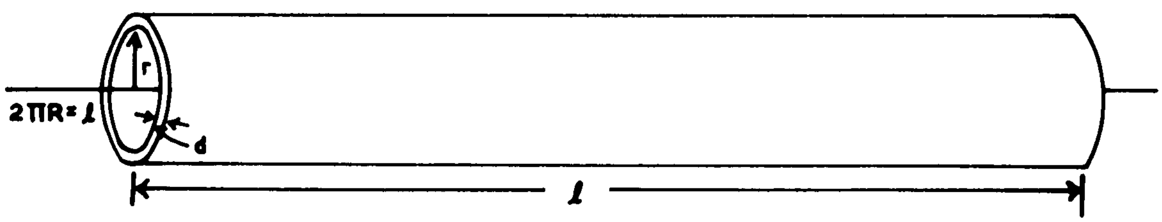
If a long thin strip of rubber or a strip of paper is bent along its length so that two ends of the strip are made to touch each other, then we get a cylindrical shell structure whose depth is equal to the width of the strip and the circumference equal to the length of the strip as shown in fig. 5.7(a). Again if the same strip of rubber or paper is bent along its width so that the two edges of the strip are made to touch each other then we get another cylindrical shell structure whose depth will be equal to the length of the strip and the circumference equal to the width of the strip, as shown in fig. 5.7(b). But if we bend the strip in skew by holding two ends of the strip, such that the axis of bending is inclined at a certain angle to the length of the strip, then the type of structure obtained will not be the same as the other two types mentioned above but it will be as shown in fig. 5.8(a). If bending in the rest of the length of the strip is continued, keeping the axis of bending the same and also the radius of curvature of bending as constant, then we get a structure as shown in fig. 5.8(b) and (c). This structure resembles the structure of the wrapped part Fig. 5.4(a).

A short section of the wrapped structure is shown in Fig. 5.9. In order to maintain this configuration, torque must be applied along AB and CD to balance the bending moment. The bending moment M is given by YI/ρ, where Y = bending moment.

FIG. 5.7.



(a) BENDING OF STRIP ALONG THE LENGTH.



(b) BENDING OF STRIP ALONG THE WIDTH.

FIG. 5. 8.

BENDING OF STRIP AT SKEW.

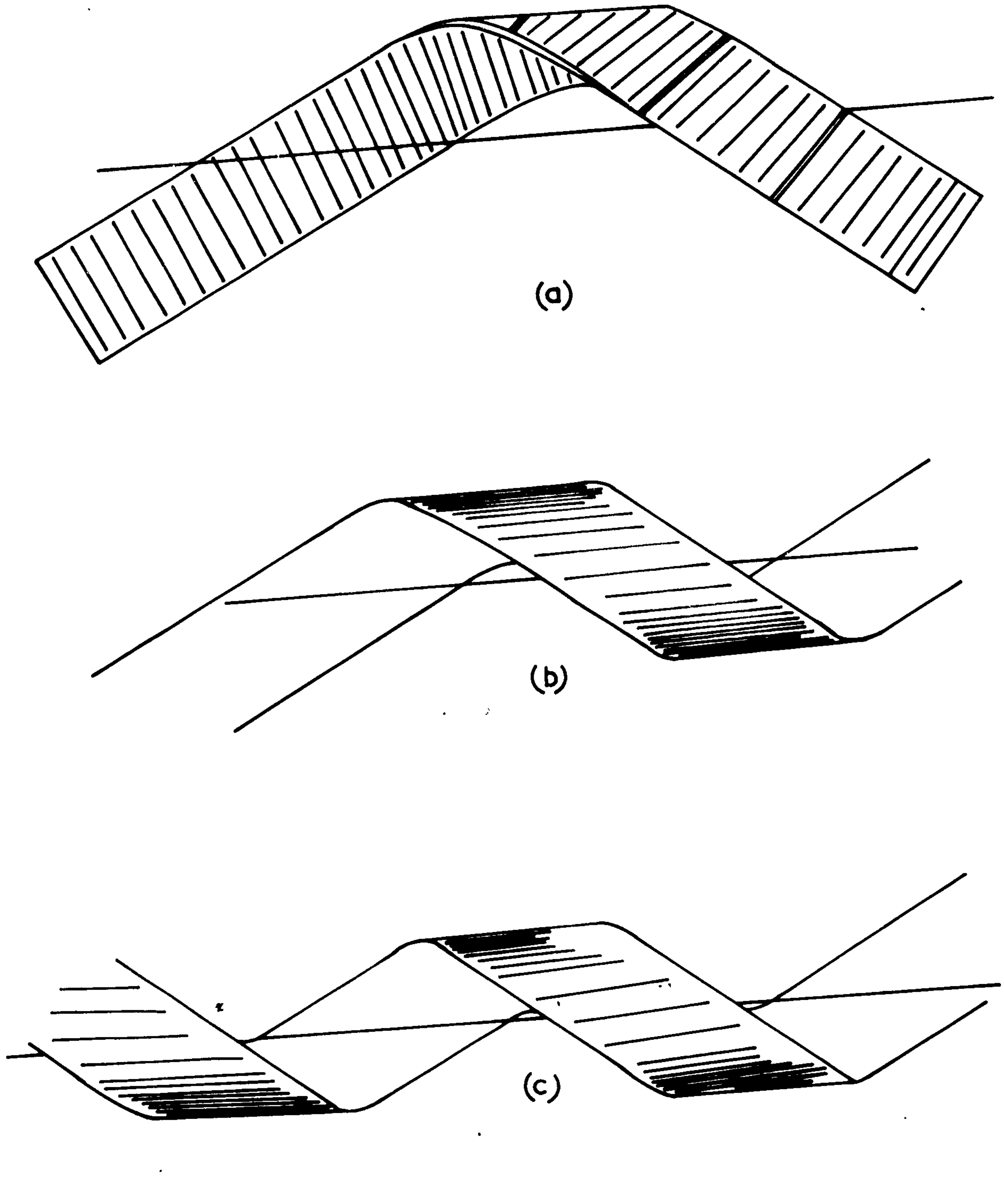
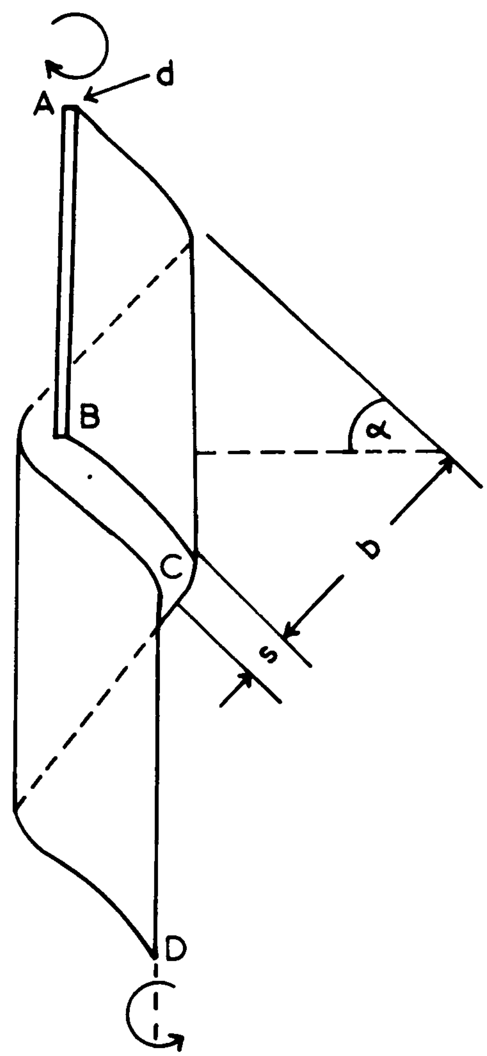


FIG. 5.9.



I is the moment of inertia of the bar about the neutral axis and ρ is the radius of curvature of the arc into which the bar has been bent.

Applying the same formula for our wrapped structure, we have

$\rho = R_2$ (radius of the wrapped structure)

If zz^1 be the neutral axis, then from Figure 5.10(c) moment of inertia of the rectangular strip which is the thickness of the rubber along AB in Figure .

$$I_{zz^1} = \int_{-d/2}^{+d/2} y^2 \frac{b}{\sin\alpha_2} \cdot dy.$$

$$= \frac{1}{12} \cdot \frac{bd^3}{\sin\alpha_2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5.16)$$

Thus Bending Moment $= M = \frac{1}{12} \cdot \frac{bd^3}{\sin\alpha_2} \cdot \frac{1}{R_2} \cdot Y$ .... (5.17)

and Torque $= \Gamma =$ Bending Moment $M = \frac{Ybd^3}{12} \cdot \frac{1}{\sin\alpha_2} \cdot \frac{1}{R_2}$ ..... (5.18)

Substituting the value of $\sin\alpha_2$ from equation (5.8) in equation (5.18) we get:

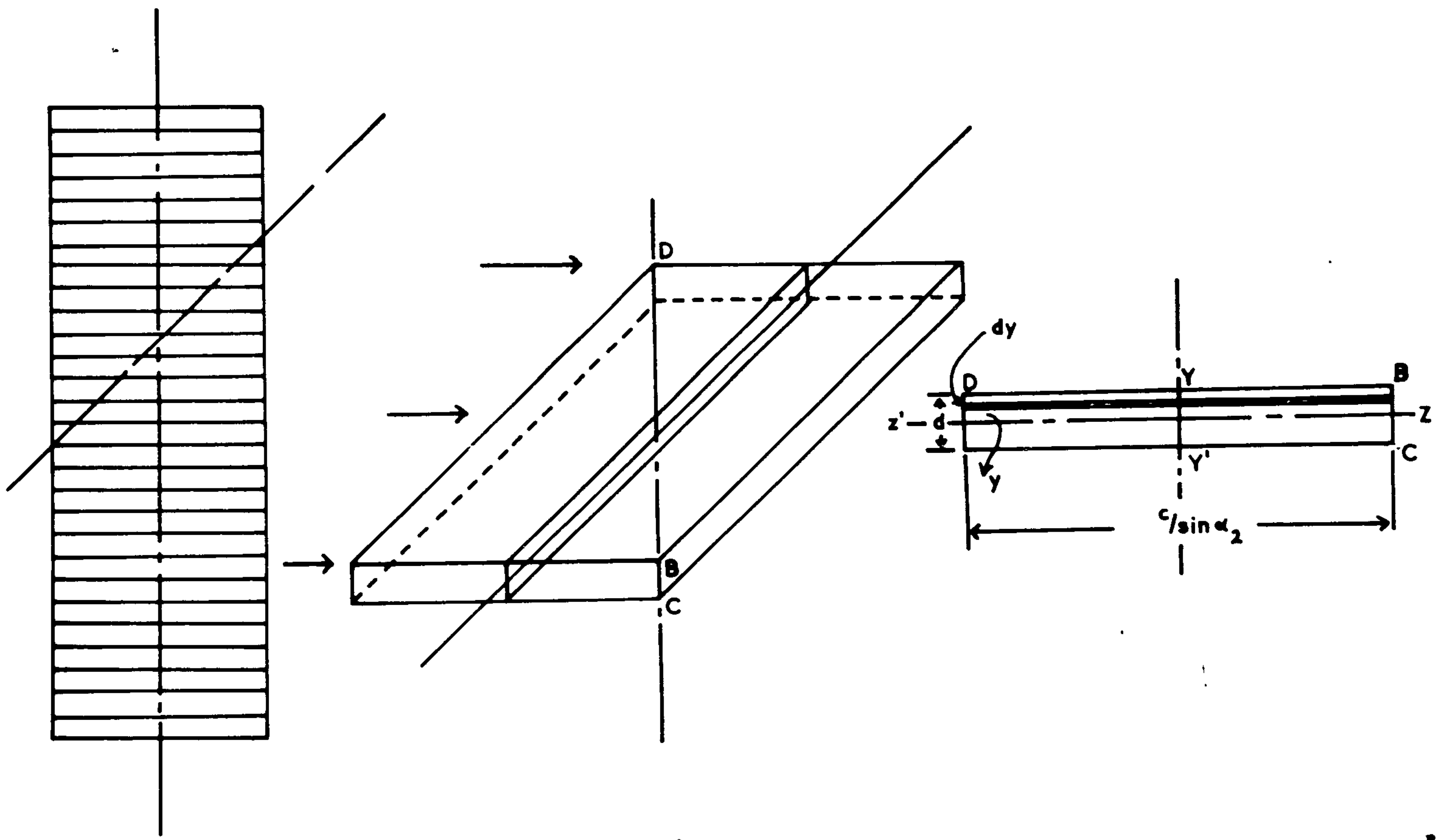
$$\Gamma = \frac{Y.b.d^3}{12} \cdot \frac{\ell_2}{2\pi R_2 n_2} \cdot \frac{1}{R_2}$$

$$= \frac{Ybd^3}{24\pi} \cdot \frac{\ell_2}{n_2} \cdot \frac{1}{R_2^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5.19)$$

Substituting the value of R_2^2 from equation (5.11) in equation (5.19)

FIG. 5. 10.

BENDING OF A BLADE IN SKEW.



(a) (b) (c)

we get:

$$\Gamma = \frac{Ybd^3}{24\pi} \cdot \frac{\ell_2}{n_2} \cdot \frac{8\pi^2 n_2^2}{\ell_2^2 \pm \ell_2\sqrt{\ell_2^2 - 4(b+s)^2 n_2^2}} \quad \ldots\ldots\ldots\ldots (5.20)$$

$$\text{or } \Gamma = \frac{Ybd^3}{3} \cdot \frac{\pi n_2}{\ell_2 \pm \sqrt{\ell_2^2 - 4(b+s)^2 n_2^2}} \quad \ldots\ldots\ldots (5.21)$$

The two solutions correspond to the two forms having the same values of ℓ_2, n_2, b and s being different values of α_2

Strain energy

When a strip is bent by an externally applied couple work is done on it. This work is stored in the body in the form of energy, and is known as the strain energy due to bending. Strain energy due to bending is given by half the product of the bending moment and the angle which the ends of the bent strip submit at the centre of the circle of which it is a part. In our case, as the bending moment is equal to the torque, the strain energy stored in the wrapped part will thus be:

$$E_{wr} = \frac{1}{2} \text{ torque x total twist}$$

$$= \frac{1}{2} \times \Gamma \times 2\pi n_2$$

$$= \pi n_2 \Gamma \quad \ldots\ldots\ldots (5.22)$$

Special cases

(i) considering the positive sign in equation (5.21)

$$E_{wr} = \frac{Ybd^3}{3} \quad \frac{\pi^2 n_2^2}{\ell_2 + \sqrt{\ell_2^2 - 4(b+s)^2 n_2^2}} \qquad \text{..........} \quad (5.23a)$$

(ii) now if $s = -b$

$$E_{wr} = \frac{Ybd^3}{3} \cdot \frac{\pi^2 n_2^2}{2\ell_2} \qquad \text{...........} \quad (5.23b)$$

agreeing with the simple equation for the planar wrapping

(iii) when $s = o$

$$E_{wr} = \frac{Ybd^3}{3} \cdot \frac{\pi^2 n_2^2}{\ell_2 \pm \sqrt{\ell_2^2 - 4b^2 n_2^2}} \qquad \text{..........} \quad (5.23c)$$

(iv) when $s = o$ and $n_2 = \frac{\ell_2}{2b}$

$$E_{wr} = \frac{Ybd^3}{3} \cdot \frac{\pi^2 . \ell_2^2 / 4b^2}{\ell_2}$$

$$= \pi^2 . \frac{Ybd^3}{3} \cdot \frac{\ell_2}{4b^2} \qquad \text{..............} \quad (5.23d)$$

5.5 Potential energy of the system

Energy is defined as the capacity for doing work, and when a stone is at a height of hcm. above the ground its weight has the capacity of doing mgh ergs of work in virtue of its position. It is said to possess energy of position or potential energy.

When a ribbon is twisted the length along the axis of the structure is smaller than the length along the axis of the strip,

i.e. twisting is followed with contraction in length. If a tension had been acting on the ribbon when it was being twisted, or if we imagine that a load was hung from one end of the ribbon and twist was imparted from the other end, the load will be lifted up due to the contraction in length. Thus work is being done by the load or the tension during the process of twisting, which is stored as potential energy of the system. If the twist is taken out or the twisting process is reversed, the potential energy is liberated.

As the contraction in length is negligible (assumption (b), page 61) the potential energy due to the twisted part will be assumed to be negligible.

Potential energy due to the wrapped part

If the length along the axis of the wrapped structure be h_2 cms. for one turn of twist and the length along the ribbon be ℓ_2 cms., then the total contraction in length when n_2 number of turns have been introduced will be:

$$\text{Contraction} = \ell_2 - n_2 h_2 \qquad \text{........} \quad (5.24)$$

Now the length of 1 turn along the axis $= h_2$

$$= \left[\frac{\ell_2^2}{n_2^2} - 4\pi^2 R_2^2 \right]^{\frac{1}{2}}$$

$$= \left[\frac{\ell_2^2}{n_2^2} - 4\pi^2 \cdot \frac{\ell_2^2 \pm \ell_2 \sqrt{\ell_2^2 - 4(b+s)^2 \cdot n_2^2}}{8\pi^2 n_2^2} \right]^{\frac{1}{2}}$$

$$= \left[\frac{\ell_2^2}{n_2^2} - \frac{1}{2}\frac{\ell_2^2}{n_2^2} \cdot \left\{ 1 \pm \sqrt{1 - 4\frac{(b+s)^2}{\ell_2^2} n_2^2} \right\} \right]^{\frac{1}{2}}$$

therefore $h_2 = \frac{\ell_2}{n_2} \left[1 - \frac{1}{2}\left\{ 1 \pm \left(1 - 4(b+s)^2 \frac{n_2^2}{\ell_2^2}\right)^{\frac{1}{2}} \right\} \right]^{\frac{1}{2}}$

$$= \frac{\ell_2}{n_2} \left[\frac{1}{2} \mp \frac{1}{2}\left(1 - 4(b+s)^2 \frac{n_2^2}{\ell_2^2}\right)^{\frac{1}{2}} \right]^{\frac{1}{2}}$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\ell_2}{n_2} \left[1 \mp \left(1 - 4(b+s)^2 \frac{n_2^2}{\ell_2^2}\right)^{\frac{1}{2}} \right]^{\frac{1}{2}} \quad \ldots\ldots (5.25)$$

therefore contraction in length $= \ell_2 - n_2 h_2$

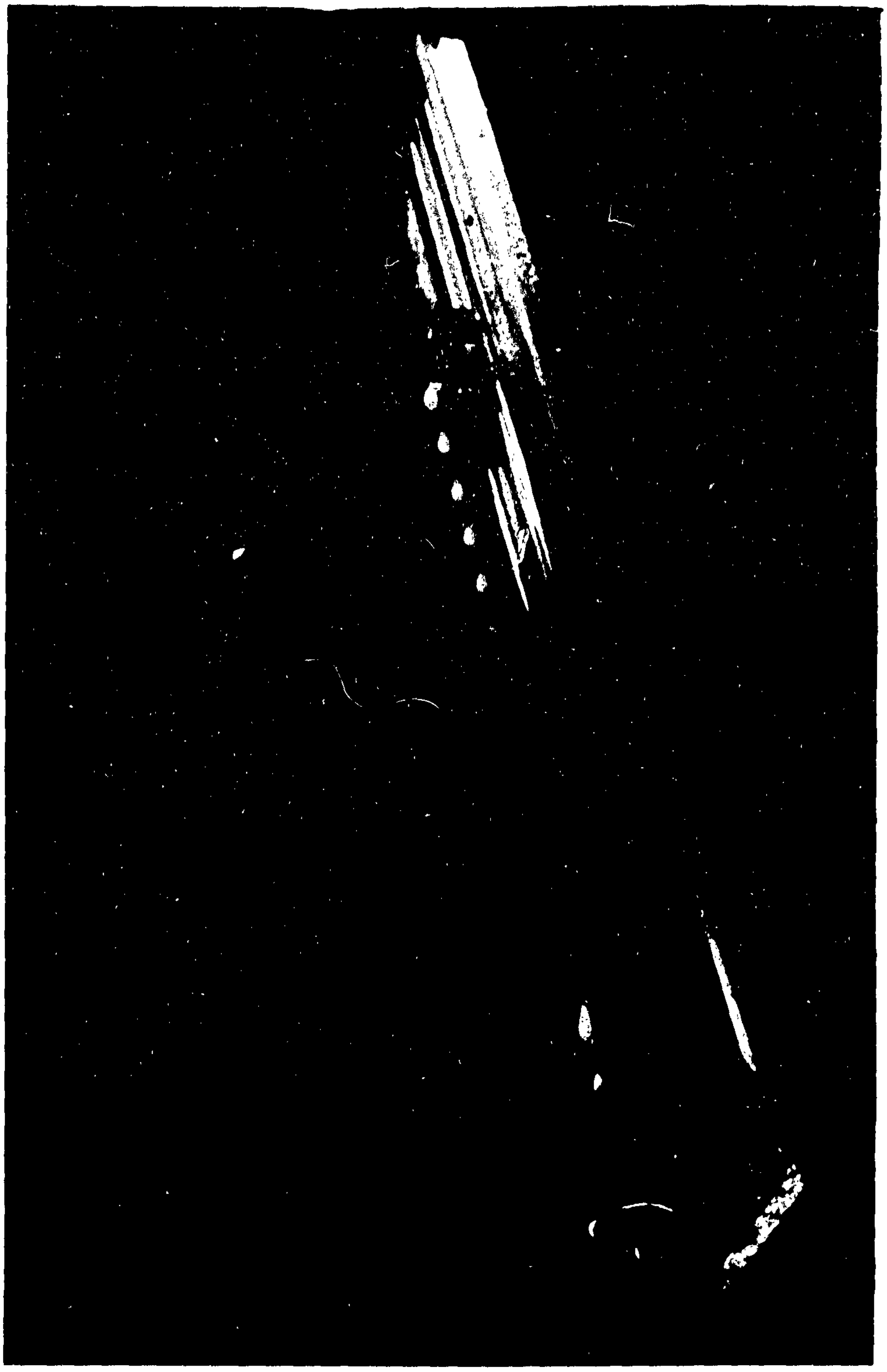
$$= \ell_2 - \frac{1}{\sqrt{2}}\ell_2 \left[1 \mp \left\{ 1 - 4(b+s)^2 \cdot \frac{n_2^2}{\ell_2^2} \right\}^{\frac{1}{2}} \right]^{\frac{1}{2}}$$

$$= \ell_2 \left[1 - \frac{1}{\sqrt{2}} \left\{ 1 \mp \left(1 - 4(b+s)^2 \cdot \frac{n_2^2}{\ell_2^2}\right)^{\frac{1}{2}} \right\}^{\frac{1}{2}} \right] \text{cms.} \quad \ldots (5.26)$$

Therefore if W be the tension during the twisting process or if W be the weight in gms. hung from the end of the ribbon during twisting:

Potential Energy $= W . \ell_2 \left[1 - \frac{1}{\sqrt{2}} \left\{ 1 \mp \left(1 - 4(b+s)^2 \cdot \frac{n_2^2}{\ell_2^2}\right)^{\frac{1}{2}} \right\}^{\frac{1}{2}} \right]$ gms.cm.

$\ldots\ldots\ldots\ldots\ldots\ldots (5.27)$



Special cases

(i) Considering the negative sign in equation (5.27) corresponding to the positive sign in equations (5.11) and (5.21)

therefore Potential Energy

$$= W\ell_2 \left[1 - \frac{1}{\sqrt{2}} \left\{ 1 - \left(1 - 4(b + s)^2 \frac{n_2^2}{\ell_2^2}\right)^{\frac{1}{2}} \right\}^{\frac{1}{2}} \right] \text{gms.cm.} \quad \ldots\ldots \ (5.28a)$$

(ii) now if $s = - b$

Potential Energy $= W\ell_2$ .... (5.28b)

In the planar wrapped structure, the whole length of the strip is taken up.

(iii) when $s = 0$

$$\text{Potential Energy} = W\ell_2 \left[1 - \frac{1}{\sqrt{2}} \left\{ 1 - \left(1 - 4\frac{b^2 n_2^2}{\ell_2^2}\right)^{\frac{1}{2}} \right\}^{\frac{1}{2}} \right] \text{gms.cm.}$$

.... (5.28c)

(iv) when $s = 0$ and $n_2 = \frac{\ell_2}{2b}$

Potential Energy $= W\ell_2 \left(1 - \frac{1}{\sqrt{2}}\right)$ .... (5.28d)

5.6 Combination of Twisted and Wrapped forms

We can now turn to a numerical assessment of the energy relations by substituting the appropriate dimensional moduli for the rubber strip, as reported in last chapter.

Figures 5.11 and 5.12 show the energy values for various values of twist corresponding to the twisted form and the wrapped form.

FIG. 5.11.

THEORETICAL CURVES FOR 1·0cm. WIDE STRIP

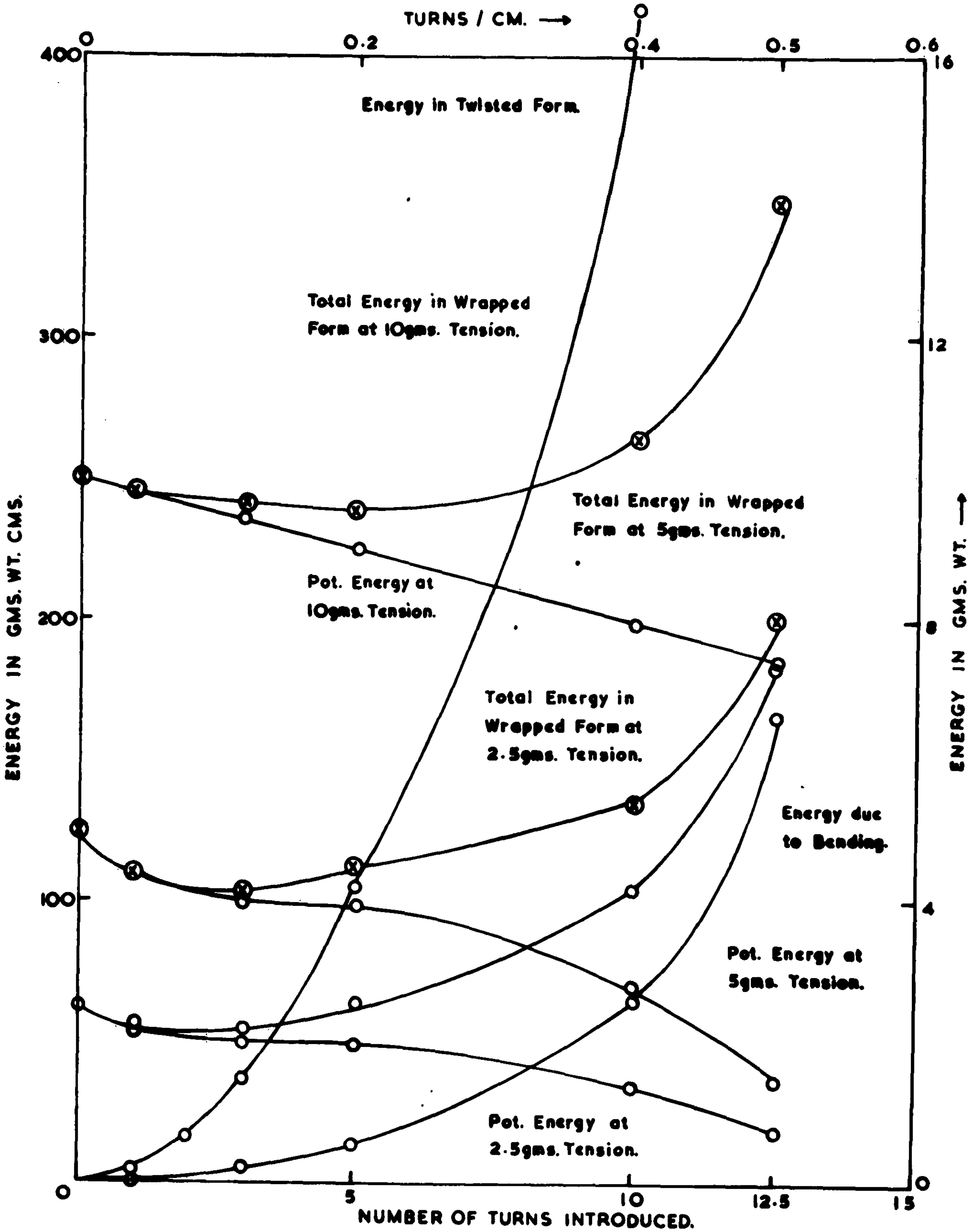
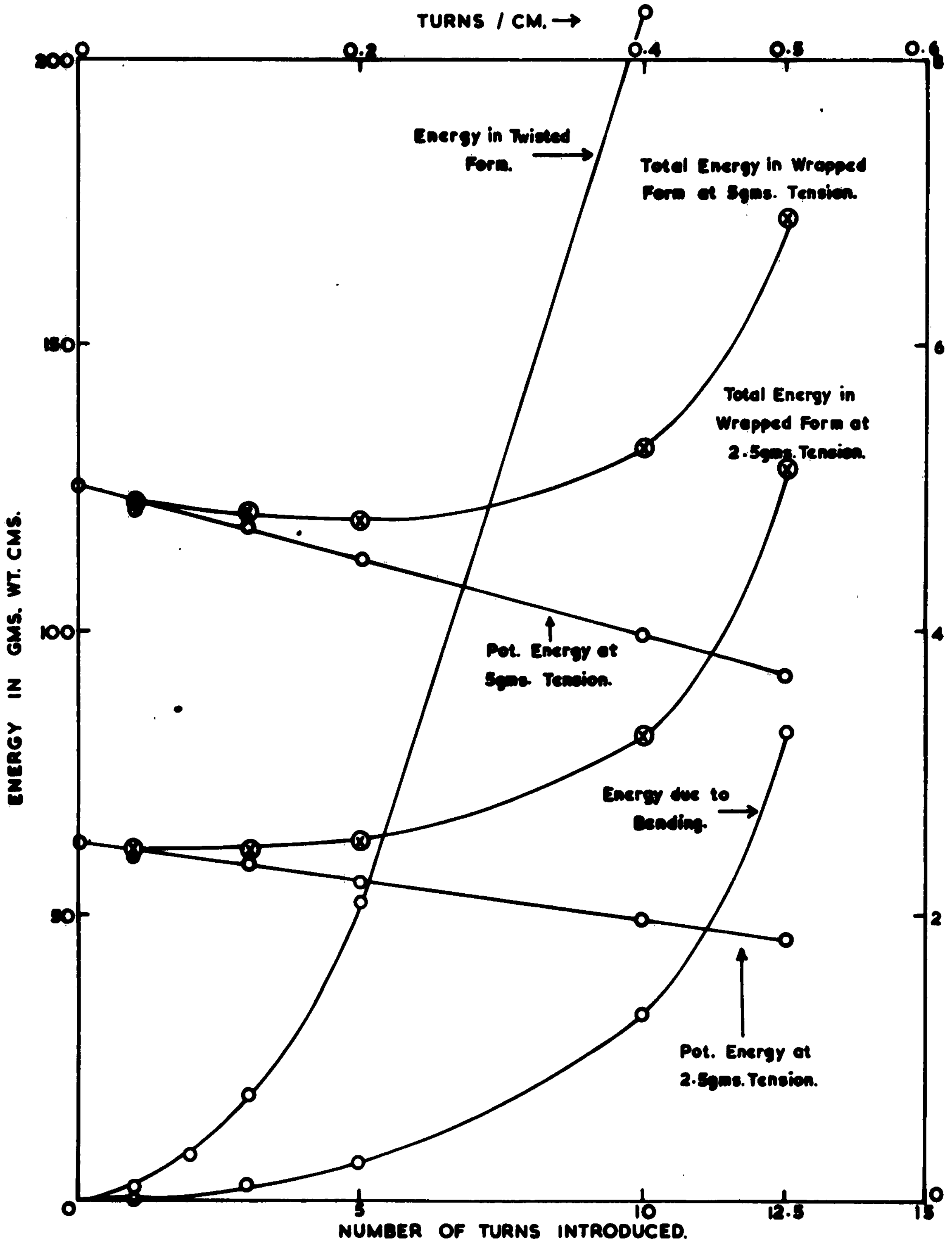


FIG. 5. 12.

THEORETICAL CURVES FOR 0.5cms WIDE STRIP.



It will be noted that the energy due to bending in the wrapped form is less than the energy due to torsion in the twisted form, indicating that the wrapped form would be favoured. However, when the potential energy in the wrapped form is added in, the situation changes.

At low twists, the twisted form has the lower energy and will thus be the stable configuration; but at high twists the wrapped form has the lower energy and will become stable.

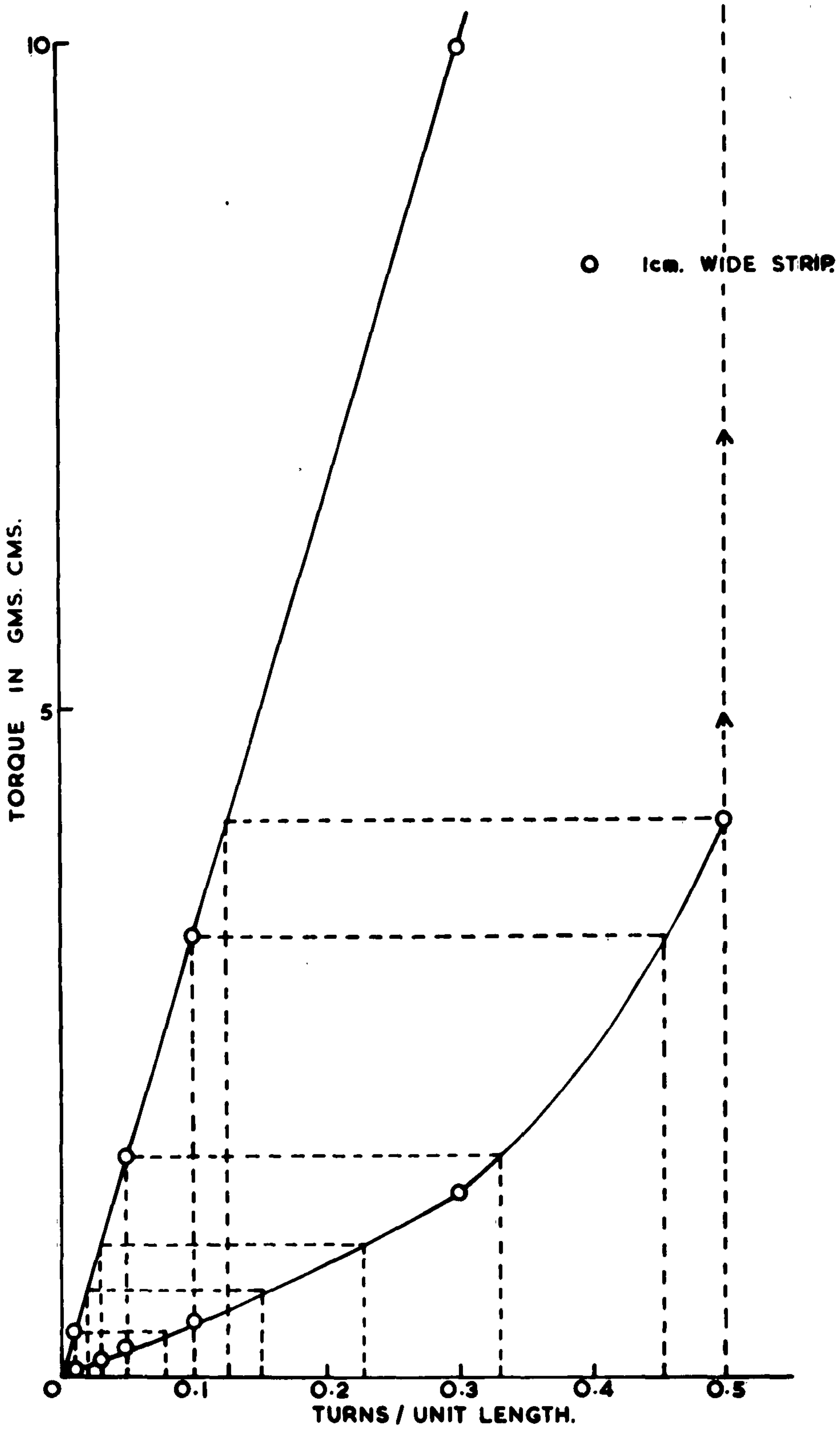
It is now necessary to consider the possibility of the twisted and wrapped forms existing together. If a given number of turns n are introduced into the strip of length l, then these may be divided in different proportions between different lengths in each form. However, stability will only be achieved when the torque in the twisted form equals the torque in the wrapped form. Thus if a given number of turns/unit length is selected from the twisted form, the number of turns per unit length in the wrapped form may be determined from the torque-twist curves shown in Figure 5.13. The division of length between the two forms then follows from the equation:

$$n = n_1 l_1 + n_2 (l - l_1) \qquad \text{............ (5.29)}$$

It will be observed from Figure 5.13 that when this torque in the wrapped part has achieved the maximum value (at 0.5 turns per cm.) any further increase in torque in the twisted part will only close

FIG. 5. 13.

THEORETICAL CURVE FOR 1cm. WIDE STRIP.



the gap between the two edges of the strip in the wrapped form, i.e. compression takes place without altering the total number of turns in the strip, which gives the jammed structure and thus the equality of torque is maintained. A small amount of energy will be involved in this change over and has been neglected from our calculations at the present stage.

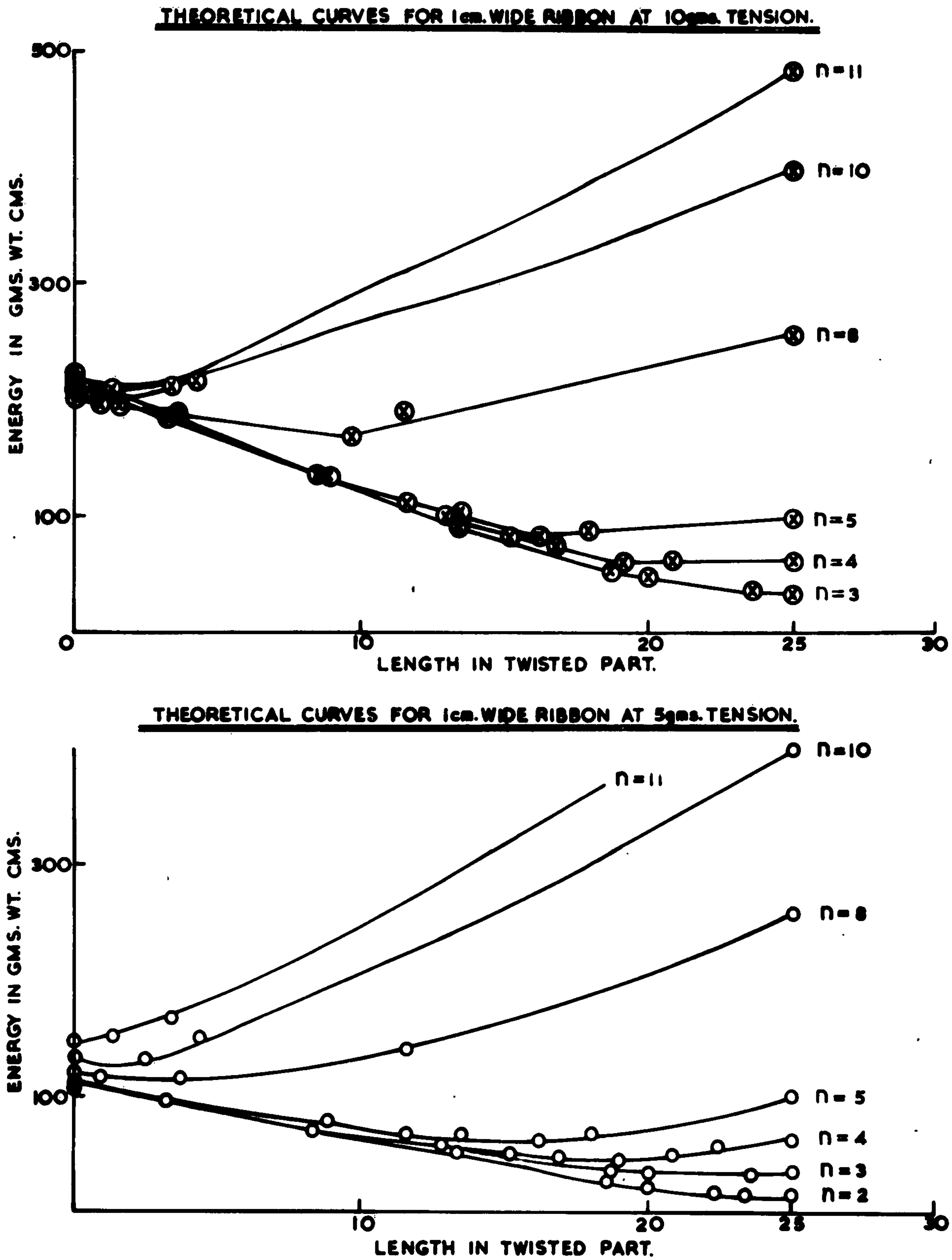
The total energy may be obtained by adding the appropriate combinations from the twisted and wrapped parts. An example of the calculation is given in Table 5.1. The curves obtained are shown in Figures 5.14(a) and (b), and 5.15. The minima in these curves represent the position of stability and so the division between the wrapped and twisted form is established. This theoretical division is shown plotted against number of turns in Figure 5.16.

When the total number of turns per cm. is less than 0.5 turns per cm., theoretically a number of forms can be available. The structure could have a combination of twisted form and wrapped form with or without gap or the whole structure may be in the wrapped form with gap between the edges. The form which will have the minimum energy will be obtained in practical cases.

Figure 5.16 is similar in form to the experimental curves, and a comparison of the two is made in Figure 5.17; the agreement is not exact and this may be due to the following causes:

(a) neglect of the energy relations at the boundary between the twisted and wrapped forms.

FIG. 5. 14.



THEORETICAL CURVES FOR 1cm. WIDE RIBBON AT 10gms. TENSION.
ENERGY IN GMS. WT. CMS.
500
300
100
0
10
20
25
30
LENGTH IN TWISTED PART.
n=11
n=10
n=8
n=5
n=4
n=3
THEORETICAL CURVES FOR 1cm. WIDE RIBBON AT 5gms. TENSION.
ENERGY IN GMS. WT. CMS.
300
100
10
20
25
30
LENGTH IN TWISTED PART.
n=10
n=11
n=8
n=5
n=4
n=3
n=2

FIG. 5. 15.

THEORETICAL CURVES FOR 0·5cm. WIDE RIBBON AT 5gms. TENSION.

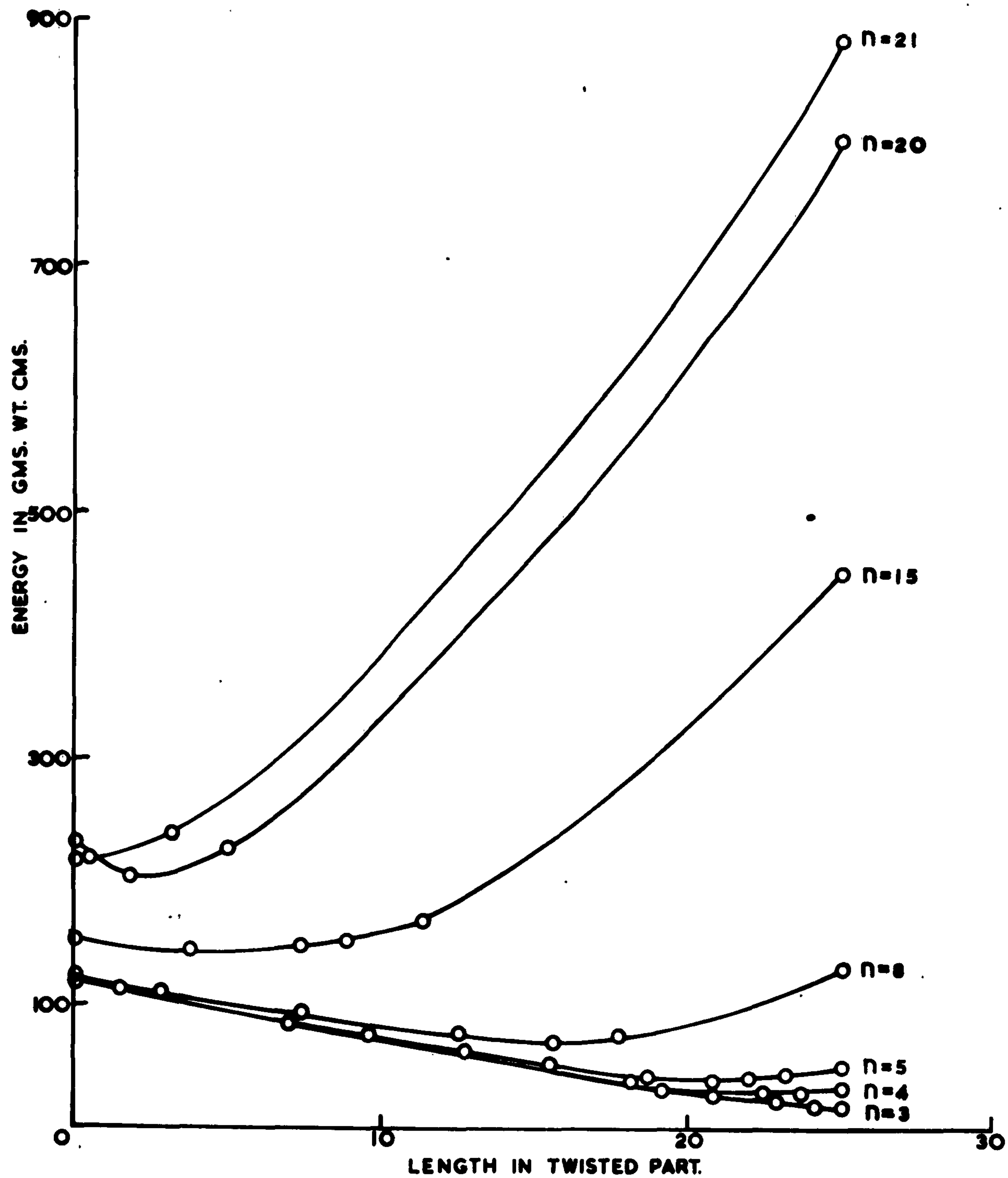


FIG. 5. 16.

THEORETICAL CURVES FOR DISTRIBUTION OF LENGTH DURING TWISTING.

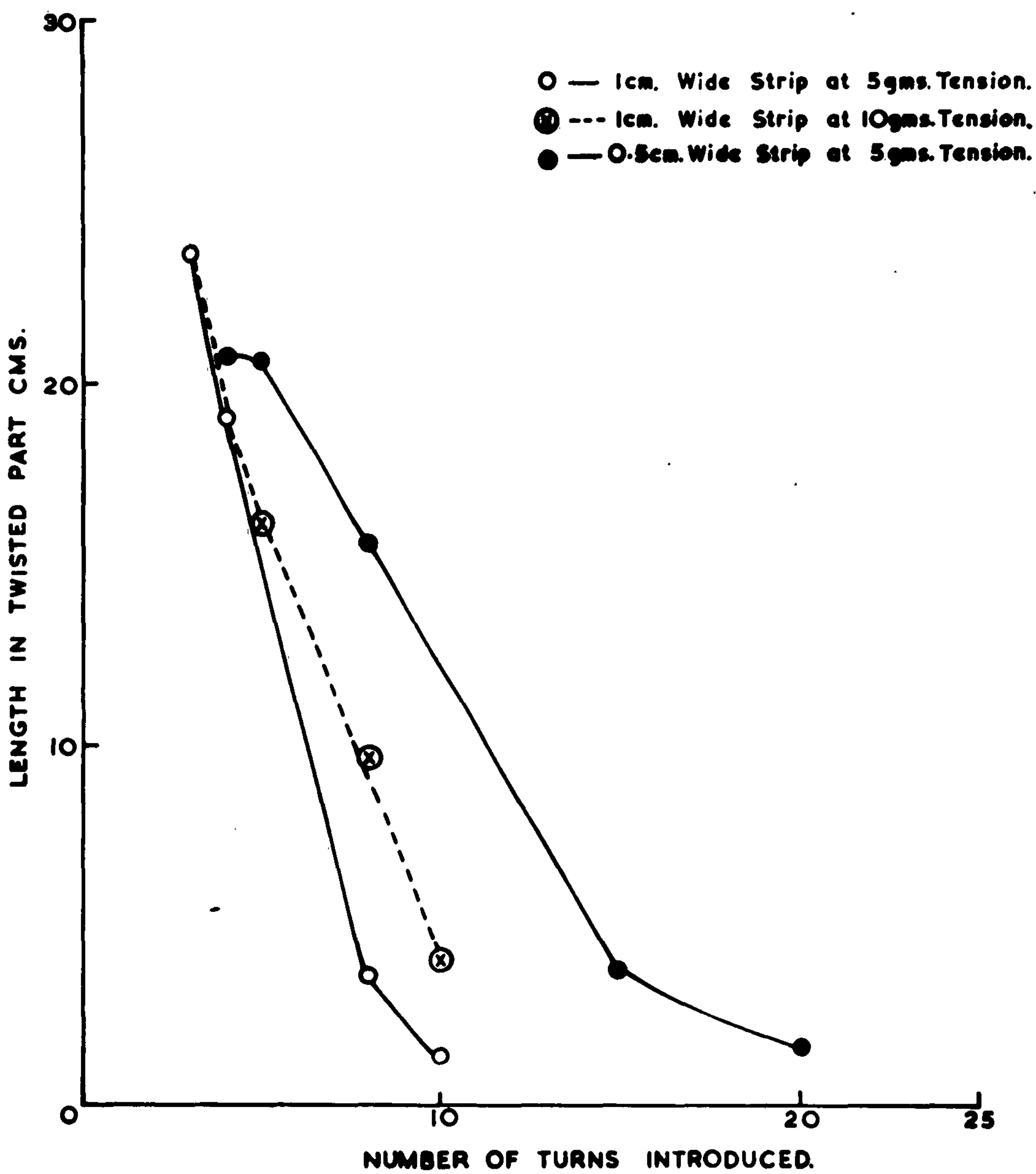


FIG. 5. 17.

COMPARISON OF THEORETICAL and EXPERIMENTAL RESULTS.

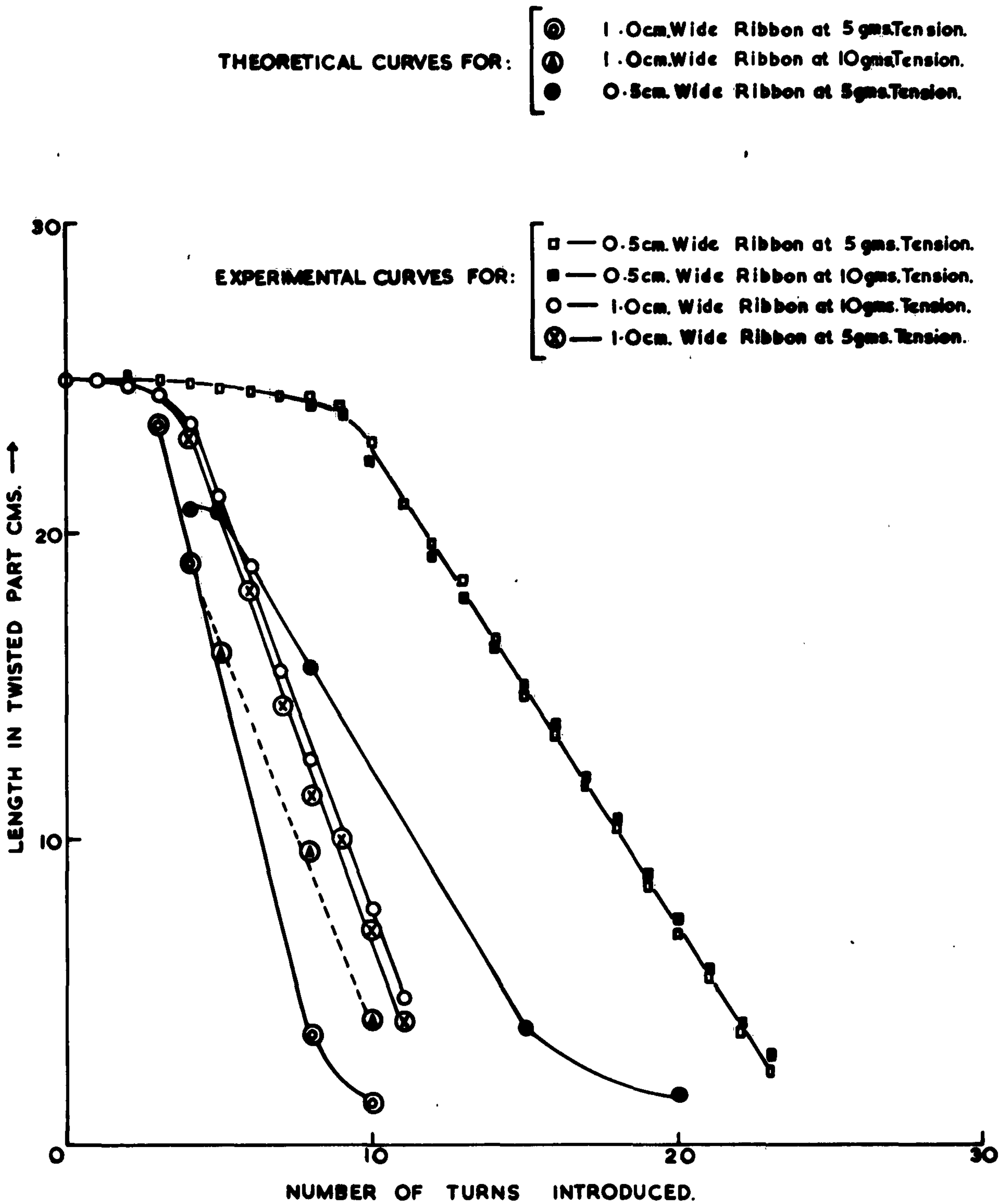


TABLE 5.1

From Eqn. $\ell_1 + \ell_2 = \ell$		From Figure 5.13		From Figure 5.11			
		For Equal Torques					
Length in Twisted Part ℓ_1 cms.	Length in Wrapped Part ℓ_2 cms.	Turns/cm in Twisted Part	Turns/cm in Wrapped Part	Energy in Twisted Part gms.wt.cms.	Energy in Wrapped Part gms.wt.cms.	Potential Energy Due to CONTRACTION gms.wt.cms.	TOTAL ENERGY gms.wt.cms.
25.00	0	0.20	-	99.97	-	-	99.97
18.06	6.94	0.10	0.46	18.05	27.74	22.01	67.81
16.17	8.83	0.08	0.42	11.60	26.51	23.03	61.15
13.52	11.48	0.06	0.36	4.86	23.90	38.62	67.39
11.61	13.39	0.05	0.33	2.80	21.92	43.34	68.16
8.84	16.16	0.04	0.29	1.42	19.33	56.44	77.19
3.20	21.80	0.03	0.23	0.29	15.34	83.79	99.42
0	25.00	0	0.20	-	13.74	99.46	113.20

Length of strip .... 25 cms.
Total number of turns introduced .. 5 turns

Twisting tension .... 5 gms.
Gap S .... 0 cm.
Ribbon width .... 1 cm.

(b) neglect of the contraction in twisted part.

(c) neglect of the effects of lateral compression of the strip due to jamming.

(d) neglect of energy relations when the gaps are closed.

(e) general assumption of small strain elastic theory in a problem involving large strains.

(f) experimental error, though the only possible source of appreciative error here is non-uniformity in the rubber strip leading to inappropriate values of the moduli used in the calculations.

5.7 Future work

The experimental and theoretical studies have demonstrated with fair agreement how a flat rubber strip can twist in two quite distinct forms. The work now has to be extended to see how far similar ideas apply in the twisting of textile yarns. As an intermediate stage, the twisting of flat bundles of monofilaments or yarns will be studied.

We now see that there are three ideal forms of twisted structure to be examined. These are:

(a) twisted cylindrical bundle

(b) twisted flat ribbon

(c) wrapped flat ribbon.

The differences in filament path and in the structural parameters of yarns in the three forms will have to be established, together with an appreciation of their effect on yarn properties.

Appendix : Personnel and Expenditure

The work described in the report was carried out by Mr. A. J. Booth, B.Sc.Tech. and Mr. O. N. Bose, B.Sc.,B.Sc.Tech. under the supervision of Dr. J. W. S. Hearle, M.A.,Ph.D.,F.InstP,F.T.I.

The man-hours spent on the project were approximately 2,000 hours by Mr. Booth, 2,000 hours by Mr. Bose (supported only in part from contract funds) and 200 hours by Dr. Hearle, plus the services of typists and laboratory and workshop technicians.

Material costs were in the region of £200.

The work forms part of a wider programme of work on the mechanics and structure of twisted yarns, being carried on under the direction of Dr. J. W. S. Hearle